Iroquois

NEW STANDARD ARITHMETICS

GRADE EIGHT

DE GROAT AND YOUNG

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IROQUOIS
NEW STANDARD
ARITHMETICS

GRADE EIGHT

by

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for
The State of New York

IROQUOIS PUBLISHING COMPANY, Inc.
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THE IROQUOIS NEW STANDARD ARITHMETICS

By

DeGroat and Young

THREE-BOOK SERIES

BOOK ONE, Grades Three and Four

BOOK TWO, Grades Five and Six

BOOK THREE, Grades Seven and Eight

SERIES BY GRADES

Grade Three	Grade Six
Grade Four	Grade Seven
Grade Five	Grade Eight

IROQUOIS PROTRACTOR, RULER, AND SQUARE

A useful combination drawing instrument for geometric constructions and measurements. Consists of an angle protractor and four different rules. Edges die-cut at right angles for drawing perpendiculars.

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To Teachers of Arithmetic:

The IROQUOIS NEW STANDARD ARITHMETICS present the subject so simply and clearly that the pupil easily understands basic principles, methods, and applications. The wealth of activities and appealing problems, built around normal pupil interests in the home, school, and community, gives further incentive for mastering the subject.

1. Authors. The authors are intimately acquainted with the problems and methods of arithmetic teaching in all the grades.

Dr. DeGroat is a recognized authority in the field. He has built numerous elementary courses of study, including arithmetic. He has directed important researches and investigations in arithmetic and is co-author of an exceptionally successful series of arithmetics.

Dr. Young is a specialist in elementary education. His work in this field has given him firsthand knowledge of the problems and methods of arithmetic teaching. His grasp of social problems, as they affect the child, has been invaluable in preparing the problems and activities.

2. Aims. Before the IROQUOIS NEW STANDARD ARITHMETICS were written, an exhaustive study was made of the findings of the authoritative modern researches in arithmetic and of new courses of study of recognized value. As a result of these studies, the authors have: (1) stepped up certain basic topics taught in the grades; (2) made provision for a longer span of time for mastery of the fundamentals; (3) graded the various topics to accord with their usefulness to the child; (4) provided enriching materials in accordance with the special recommendations of leading courses of study.

3. Organization. The books have been carefully and systematically organized to provide for simple and clear presentation of each topic, adequate drills, maintenance of skills already acquired, periodic tests, and self-rating charts.

(a) Processes. A most effective plan of process development has been followed whereby *but one step is taught at a time.* The meaning and analysis of each process is given step by step.

The entire series of process developments forms a *complete method course* in arithmetic. For ready reference by both teacher and pupil, the step-by-step development of each process is boxed in to make it stand out boldly. So clearly and simply built are these process developments, so carefully graded are the steps in the presentation of each process, that the books are self-teaching. No teacher's manual is required.

(b) Drill According to Difficulty. Research has definitely shown that the number combinations in addition, subtraction, multiplication, and division vary greatly in difficulty. Throughout this series, the 25 hardest combinations in each operation are drilled four times as often as the 25 easiest. The combinations of intermediate difficulty are drilled proportionately.

(c) Problem Study. Problem study, presented in an orderly progression throughout the books, teaches the pupil to reason correctly and to work systematically in solving problems.

(d) Tests and Remedial Work. An effective and systematic testing program is provided. At suitable intervals, following each related group of new process developments, there are diagnostic tests on these processes, accompanied by the proper remedial drill. By the use of these, the pupil himself is encouraged and enabled to discover and correct his weaknesses. Self-rating progress tests are given periodically to test general progress and reasoning. There are also special tests of many types.

(e) The Index. Most complete. Every text reference has been included, classified, and cross-indexed wherever practical.

4. Content of the Seventh and Eighth Grades. The mathematics of the seventh and eighth grades is essentially the same as in the regular junior high school courses for these grades. Arithmetic remains the basic work of these years. The aim of the

Iroquois New Standard Arithmetics, therefore, is to complete in the seventh and eighth grades the presentation of the tools of arithmetic that the pupil needs in modern life, to maintain and further develop manipulation skill, and to acquaint the pupil with a broader range of social situations to which arithmetic tools are applied.

The Iroquois New Standard Arithmetic for the seventh grade is a complete arithmetic to the limits set for that grade, while the text for the eighth grade covers *the entire field* of elementary arithmetic.

The unit plan of organization is used in the seventh and eighth grades, for in a complete treatment of the processes of arithmetic at this grade level the application of these processes to modern situations is made much more effective if all related material is grouped.

(a) Re-treatment of Basic Arithmetic. The first two units in the seventh grade and the first unit in the eighth grade are devoted to a condensed, but thorough, re-treatment of the basic arithmetic developed in the earlier grades. Thereafter, manipulation skill is retained and extended by reviews and drills, and by the use of numbers in new applications.

(b) Percentage. Percentage is recognized as a subject of fundamental importance in the arithmetic of the higher grades. The subject has been partially developed in the sixth grade. In both the seventh and the eighth grades, a complete unit is devoted to the full development of percentage as an arithmetic tool.

(c) Arithmetic Applied to Practical Situations. In the earlier grades, the applications have dealt with simple situations involving school, home, and play activities of the child. In the seventh and eighth grades, a broader range of situations of vital interest to the home and to the community is presented. In the seventh grade, there are units on Arithmetic in the Home, Using

Per Cents in Business, and Money at Work. In the eighth grade, there are units on Per Cents in Everyday Life; Banking, Interest, and Money; Insurance: Protection Against Loss; Taxes: How We Support Public Services; Putting Money to Work—Investments; and How a Family Uses Arithmetic—including the buying of a home and purchases on the installment plan.

(d) Geometric Constructions and Measurements. Certain geometric concepts and principles have common applications in life. For this reason, the seventh grade includes a unit on Lines, Angles, and Geometric Figures and a unit on Scales, Areas, and Volumes. In the eighth grade, a unit on Geometric Figures and Measurements reviews and extends geometric concepts. There is also a unit on Comparing and Using Geometric Figures.

(e) Algebra. In the seventh grade, the pupil gains his first acquaintance with algebra, not by name, but by the use of letters in simple formulas. This usage is continued and extended in the eighth grade. In addition, a unit on Algebra gives a very simple, clear, and effective presentation of literal numbers and simple algebraic equations.

(f) Graphs. The modern use of graphs for the presentation and comparison of number facts is emphasized. The pupil was introduced to simple graphs in the earlier grades. In the seventh grade, an interesting and suggestive unit is devoted to the subject, covering different types. Graphs are then used as tools of arithmetic throughout the eighth grade.

(g) A Final Checkup. In both the seventh and the eighth grades, the final unit is devoted to expertly organized tests covering the work of the grade. The final unit of the eighth grade is a complete checkup of arithmetic, carefully built after a study of recognized standard tests and those given by cities and states.

5. Language. The language of the NEW STANDARD ARITHMETICS is simple, direct, clear, concise, and fitted to the pupil's

stage of development. Therefore, errors in reasoning and computation, due to language difficulties, are not apt to occur.

6. Oral and Written Work. Pupils vary greatly in the capacity to work mentally. Consequently, the authors have not drawn too strict a line between what should be written and what should be oral. The gradation of exercises and the inclusion of simple problems at frequent intervals supply material for oral work, and permit adjustment to the needs and capacities of the pupils.

7. Life Relationships. Special thought has been given to the use of problem materials which will prepare the pupil for present and future number experiences. The concrete problems are live, interesting, and practical. An exceptional feature consists of groups of socialized problems built around pupil interests and around projects based on home, farm, business, and industry.

8. Individual Differences. Recognizing the important fact that individual abilities vary, these books give challenging and enriching work to the talented pupil while providing for all the needs of the average and the slow pupil.

9. Mechanics. The books combine clarity and beauty of type and make-up with high quality and serviceability of paper and binding. The illustrations and cover design set a new standard in beauty, simplicity, and child interest.

* * *

Rich in content, attractive in make-up, and in harmony with the most authoritative modern studies in the arithmetic field, the IROQUOIS NEW STANDARD ARITHMETICS will successfully meet any valid test to which they are subjected. Between their covers, the pupil can explore with increasing knowledge and delight.

IROQUOIS PUBLISHING COMPANY, INC.

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THE SPIRIT OF SERVICE

Wind, sleet, and snow, hour after hour. Telephone poles are down, and wires have broken away. Business is interrupted, news service stopped, and urgent phone calls cannot be made. In spite of slippery roads and bitter cold, a telephone crew has rushed to the scene to repair the damage.

EIGHTH GRADE

UNIT I. ARITHMETIC IN OUR DAILY LIVES

THE TELEPHONE — A GREAT TIMESAVER

The telephone in the Morgan home was very busy one morning. Mr. Morgan called several business men and arranged for them to meet him at his office. Mrs. Morgan called the grocer and the butcher. Joan phoned a friend to meet her at the library, and Jack called his chum to make plans for the afternoon. Several times, too, other people called the Morgans.

"Think of it," said Jack, "we can use the telephone twenty-four hours every day for only $4.25 a month."

1. Dayton, where Jack lives, is a city of 200,000 people. There are 49,166 telephones in use. Round to the nearest ten-thousand the number of telephones in use, and find the average number of people for each telephone.

2. In 31 days, 10,704,300 local calls were handled in Dayton. What was the average number of calls per day?

3. In 1880, four years after the first telephone conversation in the United States, there were 30,872 telephones in use in our country. Ten years later, the number was 211,503. Find the increase in the number of telephones and the average yearly increase.

4. Recently, the United States had 19,900,000 telephones in use. This was about $\frac{1}{2}$ of the number in use in the world and $2\frac{1}{2}$ times as many as in Great Britain, France, and Germany together. About how many telephones were in use in the world? In France, Germany, and Great Britain?

300 WHAT DO YOU REMEMBER ABOUT NUMBERS?

1. Tell what kind of number each of these is: 15, XXIV, $3\frac{1}{2}$, 5 ft. 4 in.

2. Read these numbers; then write them in words: 901; 3003; 7,800,800; 10,462,079; 2,511,084,370.

3. Write in figures: (a) one hundred thousand, seventy; (b) 9 billion; (c) 625 thousand; (d) fifty-six hundred.

4. Write as Roman numerals: 1602; 423; 1206; 1940.

5. Name the periods, or groups, into which the number $28,956,297,118 is divided.

6. Round the number in example 5: (a) to the nearest hundred-thousand; (b) to the nearest ten-million; (c) to the nearest billion; (d) to the nearest ten-billion.

Check to see if each of these examples is worked correctly:

7.
5297
68
47
678
58
6148(A)

8.
5200(B)
−897(C)
4303(D)

9.
6070(E)
×209(F)
54630
12140
1268630(G)

10.
109(K)
(H)54)5899(I)
54
499
486
13(L)

11. Complete the following statements:

a. In example 7, the number labeled (A) is the ___.

b. In example 8, (B) is the ___, (C) is the ___, and (D) is the ___.

c. In example 9, (E) is the ___, (F) is the ___, and (G) is the ___.

d. In example 10, (H) is the ___, (I) is the ___, (K) is the ___, and (L) is the ___.

How do you check addition? Work these examples; then check your work:

	A	B	C	D	E
1.	20589	49753	$787.98	$964.81	98309
	548	87369	189.63	55.43	899450
	41634	651	965.29	564.41	74432
	728	73946	889.67	73.08	956075
	67986	9967	138.39	981.64	88290
	759	82657	471.37	594.78	197667

How do you prove subtraction? Work these examples, and prove your answers:

2.	42255	54024	76647	843218	63019
	23816	23762	35702	219145	45986
3.	$729.27	$373.53	$826.73	$695.18	$965.42
	196.76	286.44	529.38	217.97	317.04

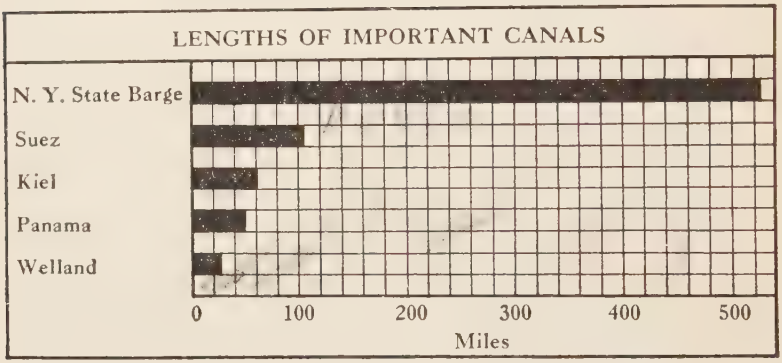
How do you check multiplication? Work the following, and check your work:

4.	5029	6380	4619	7105	4506
	32	24	46	75	98
5.	$15.91	$34.26	$72.29	$63.98	$87.32
	501	305	469	578	960

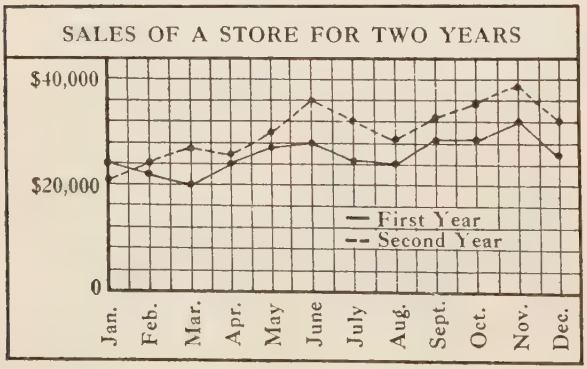
How do you prove division? Work these examples, and prove your answers:

	A	B	C	D
6.	$19\overline{)10906}$	$54\overline{)15390}$	$38\overline{)\$131.29}$	$92\overline{)\$217.12}$
7.	$45\overline{)41900}$	$79\overline{)77736}$	$63\overline{)473823}$	$36\overline{)292680}$
8.	$124\overline{)90520}$	$560\overline{)19732}$	$685\overline{)56855}$	$785\overline{)116180}$

Charts on which numbers are compared by means of lines or bars are called *graphs*.



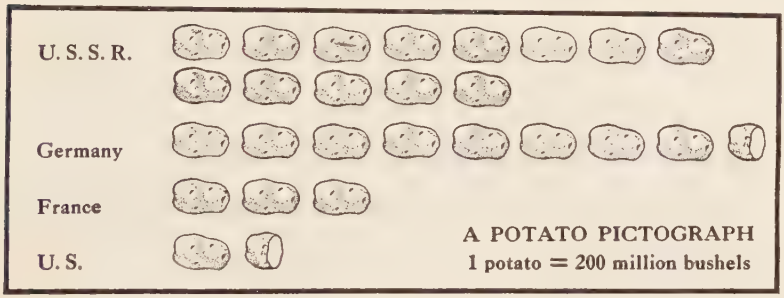
1. Tell what the above horizontal bar graph shows.
2. For how many miles does each square stand?
3. Read the graph as accurately as you can.



In this double-line graph, sales figures for two succeeding years are compared, with the amounts rounded to the nearest thousand.

4. For how many dollars does each vertical space stand?
5. About what were the total sales for each year? How much greater were the second year's sales than those of the first year?

Pictographs compare numbers by means of pictures. The pictograph below compares the potato crops of four leading potato-producing countries in a recent year.



6. Count the number of potatoes for each country, and find how many bushels of potatoes each raised.

MAKING GRAPHS

Use squared paper in building bar and line graphs. Base your scale on the largest number to be represented. If the number is 246, each square could stand for 8, and 246 would be represented by $30\frac{3}{4}$ squares. Why? How many squares would represent 246 if 1 square equaled 12? 20? 24? 48? Before comparing large numbers, round them to the nearest hundred, thousand, or ten-thousand, and so on.

1. Recently, there were 238,539 mi. of railways in the United States; 52,425 mi. in Soviet Russia; 43,128 mi. in India. Show these facts in a bar graph.

2. In vocational classes aided by federal funds, the enrollments for five recent years were: 489,900; 466,999; 503,865; 537,151; 590,892. Show this by a line graph.

3. Three states produced these amounts of maple syrup in a recent year: Vermont, 889,000 gal.; New York, 740,000 gal.; Ohio, 340,000 gal. Show these facts in a pictograph.

Before you try to work any problem, read it carefully to be sure you understand it; then follow this plan:

1. Find what the problem tells.
2. Find what the problem asks for.
3. Decide what process or processes should be used and in what order.
4. Estimate about what the answer should be.
5. Solve the problem.
6. Check your work to be sure you made no mistakes.
7. Check your answer with your estimate to see if it is reasonable.

Use the above plan in solving the following problems:

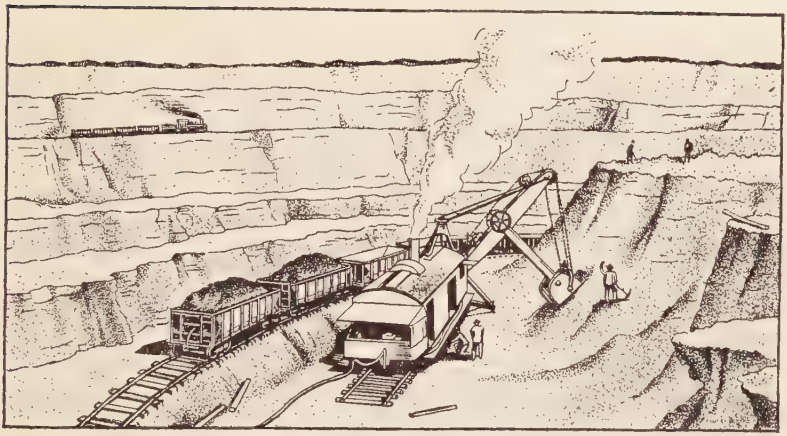
1. If 198 gal. of gasoline were used on a trip, what was the cost of the gasoline at 17¢ a gallon?

2. Find the cost of 2 doz. napkins if 6 napkins cost $1.45.

3. Mr. Rankin works from 8 A.M. to 5 P.M., with one hour out for lunch. At 60¢ an hour, how much does he earn in 5 da.?

4. The weights of the boys on a basketball team are 127, 138, 134, 119, and 129 pounds. What is their average weight?

5. James Blair drove a delivery truck 38,440 mi. in one year. He drove the truck 310 da. during the year. How many miles did he average a day?



Loading iron ore into cars.

Our country is rich in copper, gold, silver, iron, coal, and salt, which are taken from mines. Petroleum, an oil, is found in pools underground. Holes are drilled through the soil and rock to the pools, and the oil flows out or is pumped out through pipes.

1. According to recent figures, 984,323 workers in the United States are engaged in mining. About $\frac{2}{3}$ of the workers are coal miners. About how many are coal miners?

2. In a recent year, the United States produced 488,692,000 T. of coal; the United Kingdom, 224,619,000 T.; and Germany, 146,696,000 T. Show these facts by a bar graph, with the numbers rounded to the nearest million.

3. In a recent year, the following four states led in value of mineral products produced: California, $443,440,000; Pennsylvania, $617,138,000; Oklahoma, $305,152,000; Texas, $638,733,000. Find the combined value for the four states. About what fractional part was this of $4,097,942,000, the total value of our country's mineral products that year?

306 WHAT DO YOU REMEMBER ABOUT MEASURES?

1. Name 4 things measured by: (a) liquid measure; (b) dry measure; (c) weight; (d) counting; (e) linear measure; (f) surface measure.

2. Do you multiply or divide in changing smaller units of measure to larger units?

3. What process do you use in changing larger units of measure to smaller units?

See how many of the following you can answer correctly without referring to the tables of measures in the back of the book.

4. 5 bu. = ? pk. = ? qt. = ? pt.

5. $2\frac{1}{2}$ gal. = ? qt. = ? pt. = ? cupfuls.

6. 3200 oz. = ? lb. = ? cwt. = what part of a ton?

7. $\frac{1}{4}$ mi. = ? rd. = ? yd. = ? ft. = ? in.

8. 432 articles = ? gro. = ? doz.

9. $\frac{1}{8}$ T. = ? cwt. = ? lb. = ? oz.

10. 12 sq. yd. = ? sq. ft. = ? sq. in.

11. 256 pt. = ? qt. = ? pk. = ? bu.

12. 3 da. = ? hr. = ? min. = ? sec.

13. $\frac{1}{2}$ sq. mi. = ? A. = ? sq. rd. = ? sq. yd.

14. 129,600 sec. = ? min. = ? hr. = ? da.

15. What fractional part of a gallon is a pint?

16. From a barrel containing 42 gal. of insect spray, $21\frac{1}{2}$ qt. were drawn. How much spray was left in the barrel?

17. Find the cost of 7500 lb. of coal at $12.60 a ton.

What does the word **denominate** mean? Give two examples of denominate numbers; two of compound denominate numbers.

Study the following examples, and tell how each is worked:

$$\begin{array}{r} 7 \text{ bu. } 2 \text{ pk.} \\ +3 \text{ bu. } 3 \text{ pk.} \\ \hline 11 \text{ bu. } 1 \text{ pk.} \end{array} \qquad \begin{array}{r} 14 \text{ gal. } 2 \text{ qt.} \\ -9 \text{ gal. } 3 \text{ qt.} \\ \hline 4 \text{ gal. } 3 \text{ qt.} \end{array}$$

$$\begin{array}{r} 11 \text{ T. } 6 \text{ cwt.} \\ \times 12 \\ \hline 135 \text{ T. } 12 \text{ cwt.} \end{array} \qquad 15\overline{)31 \text{ A. } 65 \text{ sq. rd.}}$$

Work the following examples:

1. $\begin{array}{r} 7 \text{ gro. } 8 \text{ doz.} \\ +3 \text{ gro. } 9 \text{ doz.} \\ \hline \end{array}$

2. $\begin{array}{r} 14 \text{ hr. } 17 \text{ min.} \\ -9 \text{ hr. } 21 \text{ min.} \\ \hline \end{array}$

3. Multiply 5 yd. 2 ft. by 17.

4. Multiply 1 sq. ft. 12 sq. in. by 24.

5. Divide 35 lb. 12 oz. by 8.

6. Divide 2 da. 16 hr. by 12.

7. The distance around a square field is 6 yd. 2 ft. What is the length of each side?

8. In putting up molding in a room, Bert used 4 pieces of molding each 15′ 3″ long. How many feet and inches of molding did he use?

9. Mr. Lewis bought 3 fields containing 5 A. 120 sq. rd., 6 A. 30 sq. rd., and 15 A. 42 sq. rd. Find the total area of the fields.

10. How many hours and minutes are there between 8:45 A.M. and 12 M.? Between 1:30 P.M. and 6:15 P.M.?

Match the groups of words in Column B with the correct terms in Column A:

Column A	Column B
1. Proper fraction	a. the numerator is as large as or larger than the denominator
2. Improper fraction	b. a whole number and a fraction written together
3. Mixed number	c. the numerator is smaller than the denominator

4. Tell what each of these is: $\frac{7}{8}$, $\frac{10}{3}$, $\frac{5}{9}$, $3\frac{2}{7}$, $\frac{16}{16}$.

Study these examples, and complete the statements:

5. $\frac{2}{3} = \frac{4}{6} = \frac{12}{18}$. To change a fraction to higher terms, ____.

6. $\frac{8}{12} = \frac{2}{3}$. To change a fraction to lowest terms, ____.

7. $\frac{12}{4} = 3$. $\frac{14}{3} = 4\frac{2}{3}$. To change an improper fraction to a whole or a mixed number, ____.

8. $5\frac{1}{2} = \frac{11}{2}$. To change a mixed number to an improper fraction, ____.

9. Change these fractions to twelfths: $\frac{1}{2}$, $\frac{5}{6}$, $\frac{1}{3}$, $\frac{3}{4}$.

10. Change to lowest terms: $\frac{10}{15}$, $\frac{14}{16}$, $\frac{6}{12}$, $\frac{20}{24}$, $\frac{8}{20}$.

11. Change to whole or mixed numbers: $\frac{9}{8}$, $\frac{21}{2}$, $\frac{18}{3}$, $\frac{30}{12}$, $\frac{24}{10}$.

12. Change to improper fractions: $7\frac{2}{5}$, $11\frac{2}{3}$, $15\frac{3}{8}$, $22\frac{3}{4}$.

13. The least common denominator of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$ is 24. How do you find the least common denominator when no one of the given denominators is a common denominator?

14. Find the least common denominator of each of these fraction groups: $\frac{1}{2}$, $\frac{5}{6}$, $\frac{3}{4}$; $\frac{7}{8}$, $\frac{1}{6}$, $\frac{2}{3}$; $\frac{5}{9}$, $\frac{1}{6}$, $\frac{1}{4}$; $\frac{9}{10}$, $\frac{4}{5}$, $\frac{3}{4}$.

Tell how the example in each box is worked; then work the examples opposite it:

ADDITION OF FRACTIONS

$$4\tfrac{2}{3} = 4\tfrac{20}{30}$$
$$6\tfrac{4}{5} = 6\tfrac{24}{30}$$
$$3\tfrac{7}{10} = 3\tfrac{21}{30}$$
$$13\tfrac{65}{30} = 15\tfrac{1}{6}$$

1. $\frac{5}{8} + \frac{1}{4}$
2. $\frac{4}{5} + \frac{2}{3}$
3. $\frac{3}{4} + \frac{5}{6}$
4. $\frac{7}{8} + \frac{7}{12}$
5. $4\frac{1}{2} + 2\frac{5}{6} + 3\frac{2}{3}$
6. $2\frac{5}{6} + 4\frac{1}{3} + 9\frac{1}{4}$
7. $5\frac{7}{10} + 1\frac{1}{4} + 6\frac{3}{5}$
8. $8\frac{3}{4} + 3\frac{2}{3} + 7\frac{7}{8}$

SUBTRACTION OF FRACTIONS

$$14\tfrac{3}{8} = 14\tfrac{9}{24} = 13\tfrac{33}{24}$$
$$11\tfrac{5}{6} = 11\tfrac{20}{24} = 11\tfrac{20}{24}$$
$$2\tfrac{13}{24}$$

1. $\frac{5}{6} - \frac{1}{3}$
2. $16 - \frac{9}{10}$
3. $\frac{3}{8} - \frac{1}{6}$
4. $12\frac{9}{20} - 8\frac{4}{5}$
5. $72\frac{1}{4} - 51\frac{2}{3}$
6. $35\frac{11}{15} - 8\frac{17}{20}$

MULTIPLICATION OF FRACTIONS

$$3\tfrac{3}{7} \times 9\tfrac{1}{3} \times 3\tfrac{3}{4} =$$
$$\frac{\cancel{24}^{6}}{\cancel{7}} \times \frac{\cancel{28}^{4}}{\cancel{3}} \times \frac{\cancel{15}^{5}}{\cancel{4}} = 120$$

1. $42 \times \frac{7}{8}$
2. $\frac{9}{16} \times 72$
3. $7\frac{1}{4} \times 432$
4. $\frac{5}{6} \times \frac{14}{25} \times \frac{6}{7}$
5. $\frac{8}{9} \times \frac{2}{3} \times 1\frac{3}{4}$
6. $7\frac{1}{2} \times 8\frac{3}{4} \times 3\frac{1}{5}$

DIVISION OF FRACTIONS

$$7\tfrac{1}{2} \div 5\tfrac{5}{8} =$$
$$\frac{\cancel{15}}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{4}\cancel{5}}} = \frac{4}{3} = 1\tfrac{1}{3}$$

1. $18 \div \frac{3}{4}$
2. $\frac{9}{10} \div 6$
3. $\frac{3}{5} \div \frac{8}{15}$
4. $17\frac{1}{2} \div 5\frac{5}{6}$
5. $6\frac{2}{3} \div 3\frac{3}{4}$
6. $5\frac{1}{3} \div 2\frac{4}{5}$

7. Divide $7\frac{7}{8}$ by $\frac{7}{10}$; $8\frac{3}{4}$; $11\frac{4}{7}$; $10\frac{5}{16}$; $9\frac{11}{12}$.

Mr. Wilcox raised 112 incubator chicks, which was $\frac{7}{8}$ of the number hatched. How many chicks were hatched?

There are two ways to work this example.

Think: $\frac{7}{8}$ of the chicks = 112
$\frac{1}{8}$ of the chicks = $\frac{1}{7}$ of 112, or 16 chicks
$\frac{8}{8}$ of the chicks = 8 × 16 chicks, or 128 chicks

Or: Divide 112 by $\frac{7}{8}$. $112 \div \frac{7}{8} = \overset{16}{\cancel{112}} \times \frac{8}{\cancel{7}} = 128$

Proof: $\frac{7}{\cancel{8}} \times \overset{16}{\cancel{128}} = 112$

1. $\frac{4}{5}$ of a number is 28. What is the number? Prove.

2. 12 is $\frac{2}{3}$ of what number? **3.** 54 is $\frac{3}{4}$ of what number?

4. A merchant received 126 shirts, which was $\frac{9}{10}$ of the number he had ordered. How many shirts were ordered?

FINDING THE VALUE OF ONE OR MORE UNITS

Mr. Flynn bought $2\frac{1}{2}$ A. of land for $185. What was the price per acre?

You can work this problem in two ways.

Think: $2\frac{1}{2}$ A. or $\frac{5}{2}$ A. cost $185
$\frac{1}{2}$ A. cost $\frac{1}{5}$ of $185, or $37
1 A. cost 2 × $37, or $74

Or: Divide $185 by $2\frac{1}{2}$. $\$185 \div \frac{5}{2} = \overset{37}{\cancel{\$185}} \times \frac{2}{\cancel{5}} = \$74$

Proof: $2\frac{1}{2} \times \$74 = \frac{5}{\cancel{2}} \times \overset{37}{\cancel{\$74}} = \$185$

1. 210 = $3\frac{1}{3}$ times what number? Prove.

2. 121 ft. = $2\frac{3}{4}$ times ? ft. Prove.

3. $324 = $4\frac{1}{2}$ times ?$ Prove.

4. If $1\frac{3}{4}$ lb. of butter cost 49¢, how much will 4 lb. cost at the same rate?

1. Mrs. Kelly bought $3\frac{1}{2}$ yd. of curtain material. How many pieces, each $\frac{7}{8}$ yd. long, can she cut from this material?

2. Mr. Lothrop sprayed $7\frac{3}{4}$ sacks of a poison dust over his cotton field to kill harmful insects. Find the cost of the dust at \$3.36 a sack.

3. Janice bought $2\frac{1}{2}$ yd. of ribbon at 32¢ a yard. How much change should she receive from a dollar?

4. If boards $12\frac{3}{4}$ ft. long are cut from 14-ft. lengths, how much wood is wasted in cutting 60 boards?

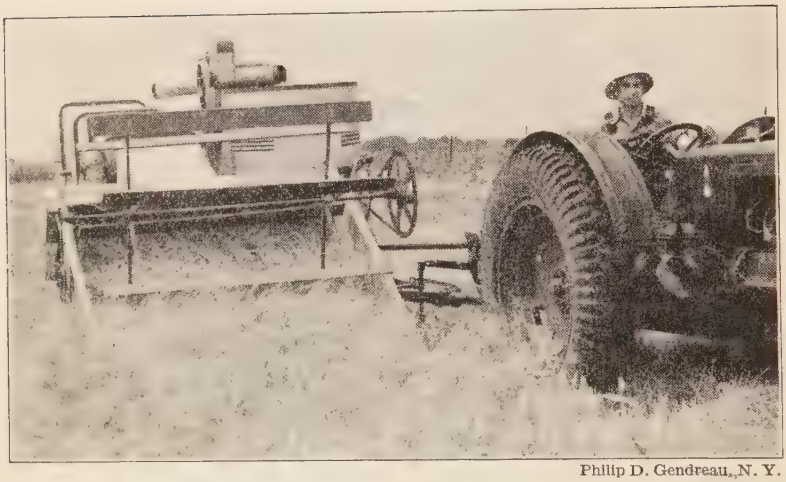
5. When bananas sell at 4 lb. for 25¢, how much will 6 lb. of bananas cost? (Count any fraction of a cent in your answer as a whole cent.)

6. When we started on our automobile ride, we had 14 gal. of gasoline in the tank. When we returned, we had $7\frac{1}{2}$ gal. left. At 18¢ a gallon, how much did the gasoline used on the trip cost?

7. A dealer bought 168 baskets of Bartlett pears, each holding $\frac{5}{8}$ bu. He paid for them at the rate of \$1.37$\frac{1}{2}$ a bushel. How much did the pears cost him?

8. A farmer received 32¢ a pound for five chickens weighing $3\frac{1}{2}$ lb., $5\frac{5}{8}$ lb., $4\frac{3}{4}$ lb., 6 lb., and $5\frac{11}{16}$ lb. How much money did he receive?

9. Make and work five fraction problems based on prices advertised in the newspapers.



Philip D. Gendreau, N. Y.

People everywhere depend upon the farmer to produce grains, fruits, vegetables, meats, and dairy products.

1. American farm incomes in a recent year totaled $8,500,000,000. The number of farms in the same year was 6,812,350. Round the number of farms to the nearest million, and find the average income per farm.

2. The amount of milk produced by American dairies in a recent year was 101,756,000,000 lb. If a pint weighs about a pound, about how many gallons of milk were produced?

3. It is estimated that the average amount of meat eaten yearly by each person in the United States is 140 lb. Our population is about 129,257,000. About how many pounds of meat do we eat in a year? About how many tons?

4. In a recent year, American farms produced the following amounts of grain: corn, 2,644,995,000 bu.; wheat, 873,993,000 bu.; barley, 219,635,000 bu.; rye, 49,449,000 bu.; oats, 1,146,258,000 bu. Show these facts in a bar graph, rounding the amounts to the nearest million.

1. Give the place value of each figure in the decimal in the box.

2. The last place to the right in a decimal gives the name of the decimal. What name is given to a decimal of 3 places? 1 place? 4 places? 2 places? 5 places?

3. Read these decimals, giving the name of each: .7, .009, 2.14, 42.8205, .032156.

Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
.	7	2	3	6	4	5

4. Round each of these decimals to the nearest tenth, the nearest hundredth, and the nearest thousandth: .2837, .0987, .30546, 56.1095, 421.5378.

5. Which is larger, .05 or .32; .273 or .269? Why?

Study each of the following examples, and then complete the statement which follows it:

6. $.25 = \frac{25}{100} = \frac{1}{4}$. To change a decimal to a common fraction, ____.

7. $.12\frac{1}{2} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{1}{8}$. To change a decimal ending in a fraction to a common fraction, ____.

8. $1.45 = 1\frac{45}{100} = 1\frac{9}{20}$. To change a mixed decimal to a mixed number, ____.

9. $\frac{2}{3} = 2 \div 3 = 3\overline{)2.00}\ .66\frac{2}{3}$. To change a fraction to a decimal, ____.

10. Change to fractions or mixed numbers: .42, .125, .084, 4.525, .28, 2.8, 8.75, .0075.

11. Change to decimals: $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{13}{20}$, $2\frac{3}{5}$, $6\frac{3}{4}$, $10\frac{11}{20}$.

What is important to remember when you copy decimals which you are to add or subtract? Copy and work the following examples; check or prove your work:

1. $68.79 + 8.85 + .6 + 76.79 + .74 + 87.57$

2. $98.8 + 66.95 + 97.3 + 2.39 + 48.69 + 6.3$

3. $75.1 + 95.5 + 3.621 + 41.5 + 3.64 + 2.861$

4. $823.6 - 157.5$ **6.** $906 - 889.2$ **8.** $68.493 - 40.945$

5. $60.67 - 31.09$ **7.** $16.5 - 8.557$ **9.** $700.34 - 30.38$

Study the following examples, and then answer the questions below:

13.65	.005	.48	.296+
×3.8	×.07	25)12.00	1.6∧).4∧750
10 920	.00035	10 0	3 2
40 95		2 00	1 55
51.87Ø		2 00	1 44
			110
			96
			14

10. In multiplying decimals, how many decimal places do you point off from the right in the product?

11. How do you divide a number by a larger number?

12. In dividing by a decimal, what changes do you first make in the divisor and the dividend? Where do you place the decimal point in the quotient?

Copy, multiply, and check:

13. 2.4×80.84 **15.** 9.83×301.7 **17.** 8.75×30.68

14. $.078 \times 274.5$ **16.** 60.9×98.86 **18.** $.97 \times 564.19$

In dividing decimals, carry every inexact division to three decimal places unless otherwise instructed. Place a + sign after the last figure in the quotient if the division is still inexact.

Copy, divide, and prove:

19. $45 ÷ 125	**22.** 4.454 ÷ .68	**25.** 47.58 ÷ 1.7
20. 698.4 ÷ .72	**23.** 9.581 ÷ 3.35	**26.** .5042 ÷ .058
21. 2.154 ÷ 286	**24.** 47.25 ÷ .084	**27.** 7.061 ÷ .943

28. Mr. Coleman sold Mr. Travis $5\frac{1}{2}$ A. of woodlot from a tract containing 36.65 A. How many acres were left?

29. Mr. Burke and his family drove from Dallas, Texas, to El Paso. The distance from Dallas to Abilene was 196.9 mi.; from Abilene to Big Spring, 108.1 mi.; from Big Spring to Pecos, 133.8 mi.; from Pecos to El Paso, 210.3 mi. How long was the trip?

30. In working out a 4-H Club project, Jack raised 246 bu. of potatoes on .8 A. of land. At that rate, how many bushels of potatoes could he have raised on 1 A.?

31. Steel bars used in a bridge construction weighed 16.45 lb. per foot. What was the weight in tons of a shipment of bars totaling 12,000 ft. in length?

32. A county accepted a bid of $143,879 for building 3.67 mi. of highway. Find the average cost per mile.

33. At $7.35 per 100 lb., how much should a farmer receive for four hogs weighing 175 lb., 204 lb., 215 lb., and 263 lb.?

Numbers may be compared by subtraction or by division. If we compare Mr. Wilson's salary of \$32 a week with Mr. Brown's salary of \$24 a week, we may say that Mr. Wilson earns \$8 a week more than Mr. Brown, or that he earns $1\frac{1}{3}$ times as much as Mr. Brown. \$32 − \$24 = \$8. \$32 ÷ \$24 = $1\frac{1}{3}$.

When two numbers are compared by division, the result is called their *ratio*. The ratio of \$32 to \$24 is \$32 ÷ \$24, or $1\frac{1}{3}$. The ratio of \$24 to \$32 is \$24 ÷ \$32, or .75 or $\frac{3}{4}$. The ratio of 24 to 12 is 24 ÷ 12 , or 2.

A ratio may be expressed as a fraction in simplest form, as a whole number, a decimal, or a mixed number.

Remember: To find the ratio of one number to another, divide the number compared by the number with which it is compared. In comparing measures, both numbers must first be expressed in the same unit of measure.

Find the ratio of:

1. 15 to 25

2. 40 to 6

3. 96 to 24

4. 9 in. to 1 yd.

5. 4 bu. to 12 pk.

6. \$3 to 60¢

7. 6.3 to 7.5

8. $3\frac{1}{2}$ to $8\frac{1}{6}$

9. $6\frac{3}{4}$ to 2.5

10. What is the ratio of the length of a pole 9 ft. long to the length of a post $4\frac{1}{2}$ ft. long?

11. During one season, a baseball team won 80 of the 144 games it played. What was the ratio of the number of games the team won to the number it lost?

12. In canning huckleberries, Mrs. Marsh uses 3 cups of sugar to each 2 qt. of berries. Using this ratio, find the amount of sugar she will use in canning 12 qt. of huckleberries.

Study these examples, and then give the short method of multiplying a whole number by 10, 100, and 1000:

$38 \times 10 = 380$ **$38 \times 100 = 3800$** **$38 \times 1000 = 38{,}000$**

The following examples show the short method of multiplying a decimal by 10, 100, and 1000. Give the rule.

$8.6 \times 10 = 86$ **$8.6 \times 100 = 860$** **$8.6 \times 1000 = 8600$**

1. Multiply by 10, 100, and 1000, using the quick way:

452 .28 31.4 \$.75 .16 ft. 200 lb. 8.129

Here are other short cuts you can use in multiplying:

To multiply a number by:

5, multiply by 10 and divide by 2.
25, multiply by 100 and divide by 4.
50, multiply by 100 and divide by 2.
$12\frac{1}{2}$, multiply by 100 and divide by 8.
$16\frac{2}{3}$, multiply by 100 and divide by 6.
$33\frac{1}{3}$, multiply by 100 and divide by 3.
125, multiply by 1000 and divide by 8.

2. $36 \times 5 = ?$
3. $84 \times 25 = ?$
4. $65 \times 50 = ?$
5. $128 \times 12\frac{1}{2} = ?$
6. $72 \times 16\frac{2}{3} = ?$
7. $18 \times 33\frac{1}{3} = ?$
8. $16 \times 125 = ?$

Find the answers by short methods:

	A	B	C
9.	36×25	13×50	$114 \times 16\frac{2}{3}$
10.	$24 \times 33\frac{1}{3}$	$32 \times 12\frac{1}{2}$	$195 \times 33\frac{1}{3}$
11.	56×125	54×5	270×125

An easy way to multiply 74 by 9 is:

$74 \times 10 = 740$ **$740 - 74 = 666$**

Multiply an easy way:

12. 625×9 **13.** \$.75 $\times$ 19 **14.** \$34 $\times$ 98

Study these examples, and then give the short method of dividing by 10, 100, and 1000:

$7.5 \div 10 = .75$ $75 \div 100 = .75$ $7500 \div 1000 = 7.5$

1. Divide by 10, 100, and 1000, using the quick way:

24 .52 71.6 \$138 \$43.98 640 ft. 2021 bu.

Here are other short cuts you can use in dividing:

To divide a number by:

25, multiply by 4 and divide by 100.
50, multiply by 2 and divide by 100.
$12\frac{1}{2}$, multiply by 8 and divide by 100.
$16\frac{2}{3}$, multiply by 6 and divide by 100.
$33\frac{1}{3}$, multiply by 3 and divide by 100.
125, multiply by 8 and divide by 1000.

2. $725 \div 25 = ?$
3. $1850 \div 50 = ?$
4. $640 \div 12\frac{1}{2} = ?$
5. $225 \div 16\frac{2}{3} = ?$
6. $1115 \div 33\frac{1}{3} = ?$
7. $2240 \div 125 = ?$

Find the answers by short methods:

	A	B	C
8.	$136 \div 25$	$680 \div 125$	$7.28 \div 50$
9.	$400 \div 16\frac{2}{3}$	$175 \div 33\frac{1}{3}$	$108 \div 125$
10.	$345 \div 50$	$79.2 \div 12\frac{1}{2}$	$29.4 \div 16\frac{2}{3}$

11. How long will 250 T. of coal last, if $12\frac{1}{2}$ T. are used each day?

12. If $33\frac{1}{3}$ A. of land cost \$2400, what does 1 A. cost?

13. If a piece of rope 625 ft. long is cut into 125-ft. lengths, how many lengths will there be?

14. Mr. Stevens bought a city lot having a frontage of 50 ft. at a cost of \$1725. How much was that a front foot?

Try to work these problems without using a pencil and paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. What fraction of a pound is 10 oz.?

2. Which is larger, .0175 or .025?

3. 24 is $\frac{2}{3}$ of what number?

4. Find the product of .5 and .05.

5. Write $1\frac{2}{5}$ as a decimal.

6. Divide 8 by .4.

7. From the sum of $\frac{1}{2}$ and $\frac{1}{4}$, subtract $\frac{3}{8}$.

8. To the product of 8 and 4, add the sum of 4 and 8.

9. If the sum of the weights of 5 people is 725 lb., what is the average weight of each person?

10. Each week at school, Irving has 135 min. of shop-work. How much time, in hours and minutes, does he spend in this work?

11. At the rate of 4 grapefruit for 25¢, how much will a dozen grapefruit cost?

12. Dave worked from 8:30 A.M. until 12 noon and received 20¢ an hour. How much did he earn?

13. Is 285 most nearly $\frac{1}{2}$ of 800, $\frac{1}{4}$ of 800, or $\frac{3}{8}$ of 800?

14. A milk dealer bought a 10-gal. can of milk and sold the milk in pint bottles. How many pints did he sell?

1. If 14 bu. 1 pk. of grapes are divided equally among 6 families, what quantity of grapes does each family receive?

2. In one county, 4-H Club boys and girls agreed to plant $321\frac{1}{4}$ A. with trees. The number of trees to be planted on each acre was 1000. How many trees were required for the project?

3. At a rate charged for regular gasoline in your town, how much would the quantity of gasoline given in this newspaper clipping cost?

It is estimated that during a recent year the average motor vehicle owner used 653 gal. of gasoline.

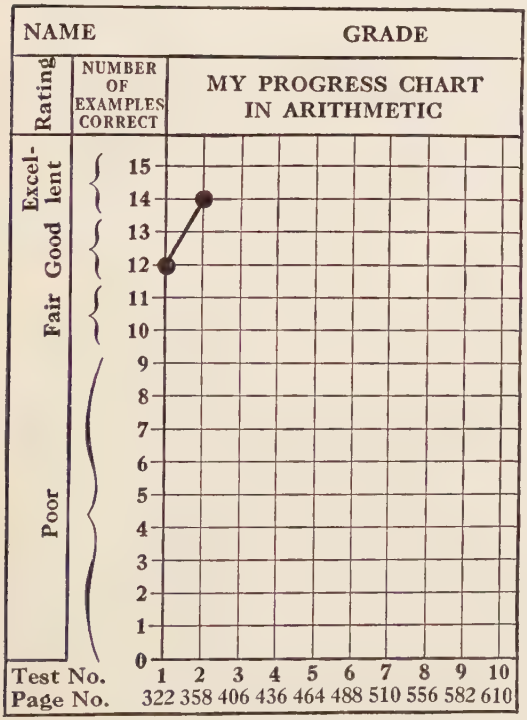
4. A plan for a machine calls for a metal bar 5.875 ft. in length. Give the length of the bar in feet and inches.

5. A county purchasing agent received bids on 600 T. of coal. The highest bid was \$4.85 a ton, and the lowest bid was \$4.32 a ton. How much per ton will he save by accepting the low bid? How much will the 600 T. of coal cost at this price?

6. Jim's marks in five arithmetic tests were as follows: 85, 90, 75, 95, 85. Construct a bar graph showing these facts.

7. Tom and Bill together worked a 4-H Club potato project. Tom worked 160 hr. on the project, and Bill worked 40 hr. Their crop was 340 bu. How many bushels of potatoes did each boy receive if they shared the crop according to the time each spent on the project?

In the eighth grade, you are going to have ten tests like the one on the following page. Show your progress in arithmetic during the year by recording your marks for these tests on a chart like the one below.



Make a large copy of this chart on cardboard. The vertical lines in the chart stand for the ten tests. Below each line are given the number of the test and its page number. The fifteen horizontal lines stand for the fifteen examples in each test.

After you have taken Test 1, place a dot where the Test 1 line crosses the line that stands for the number of examples you had right. When you know your mark in Test 2, place a dot where the Test 2 line crosses the line standing for the number of examples you had right in that test. Then connect the two dots by a line as shown above. Continue in this way until you have shown all your marks.

Try to get as high a score as possible on each test.

1.
8.508
.359
9.007
.7627
3.028
.8197

2. What is the difference between 8.745 and .764?

3. Find the product of 42.46 and .479.

4. $11\frac{2}{3} \div \frac{5}{9} = ?$

5. Divide 414.18 by 17.7.

6. Add $72\frac{3}{8}$, $26\frac{3}{4}$, 189, $379\frac{1}{2}$.

7. Multiply 79.4 by 5.07.

8. From $161\frac{3}{4}$, take $67\frac{7}{8}$.

9. $1\frac{2}{3} \times \frac{9}{10} \times \frac{2}{3} = ?$

10. A shipment of scrap iron weighed 142,865 lb. How many hundredweight did it weigh?

11. Last term, 36 of the 40 pupils in our class were promoted. Find the ratio of the number of pupils promoted to the number who were not.

12. Six clerks in a store earn weekly salaries of \$14, \$15, \$17.50, \$18, \$20, and \$21.50. Find the average wage.

13. Find the cost of 3 lb. 11 oz. of butter at 32¢ a pound.

14. A science class noted the following temperatures one June day: 6 A.M., 58°; 9 A.M., 62°; 12 noon, 70°; 3 P.M., 82°; 6 P.M., 65°. Show this information by a line graph.

15. On Monday, Wednesday, Thursday, and Friday, Tom works from 8 A.M. to 5 P.M., with 1 hr. out for lunch. Tuesday morning, he goes to part-time school, and works from 1 to 5 P.M. Saturday, he works from 9 A.M. to 12 M. Find his weekly wages if he is paid 35¢ an hour.

*Record your score on your Progress Chart. See page 321.

Gathering liquid rubber from trees in British Malaya.

Martha read in her geography text that the United States imports more than 50% of the rubber produced in the world.

The sign % is read *per cent*. Per cent means *hundredths*. 50% means 50 hundredths. $50\% = .50 = \frac{50}{100} = \frac{1}{2}$.

All per cents can be expressed as decimals or fractions. We can say that the United Kingdom imports 14%, or .14, or $\frac{14}{100}$, or $\frac{7}{50}$ of the world supply of rubber.

To change a per cent to a decimal, omit the per cent sign and move the decimal point two places to the left.

$56\% = .56$ $112\% = 1.12$ $.5\% = .005$ $\frac{3}{4}\% = .00\frac{3}{4}$ or .0075

1. Change these per cents to decimals:

4% 30% 159% $\frac{1}{2}\%$ .7% 901% 4.5% 3.06% 24.25%.

To change a decimal to a per cent, move the decimal point two places to the right and annex the per cent sign.

$.02 = 2\%$ $.00\frac{1}{4} = \frac{1}{4}\%$ $.0062 = .62\%$ $.9 = 90\%$ $1.03 = 103\%$

2. Express as per cents: .6, .82, .928, $.07\frac{1}{4}$, 3.8, .405, .04.

To change a per cent to a fraction, omit the per cent sign, write the number as the numerator of a fraction whose denominator is 100, and reduce the fraction to its simplest form.

$12\% = \frac{12}{100} = \frac{3}{25}$ $\qquad$ $175\% = \frac{175}{100} = 1\frac{3}{4}$

$$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{\cancel{75}\,^{3}}{\cancel{2}} \times \frac{1}{\cancel{100}\,_{4}} = \frac{3}{8}$$

3. Change to fractions or mixed numbers:

8% 36% 45% 72% $16\frac{2}{3}\%$ $62\frac{1}{2}\%$ $8\frac{1}{3}\%$ $\frac{1}{4}\%$ 164%

To change a fraction to a per cent, change the fraction to a decimal by dividing the numerator by the denominator. If there is a remainder and it can be written as a simple fraction, write the fraction in the quotient. Otherwise, carry the quotient to four decimal places and round it to the nearest thousandth. Express the quotient as a per cent.

$\frac{3}{4} = 4\overline{)3.00} = .75 = 75\%$ $\qquad$ $\frac{2}{3} = 3\overline{)2.00} = .66\frac{2}{3} = 66\frac{2}{3}\%$

$$\frac{7}{17} = 17\overline{)7.0000} = .4117 = .412 = 41.2\%$$

4. Change these fractions to per cents:

$\frac{5}{8}$ $\frac{3}{5}$ $\frac{11}{25}$ $\frac{3}{11}$ $\frac{5}{7}$ $\frac{7}{9}$ $\frac{13}{15}$ $\frac{9}{16}$ $\frac{7}{12}$ $\frac{11}{32}$

The following table gives the commonly used per cents with their decimal and fraction equivalents. Master these equivalents. You will find them most helpful in your arithmetic work.

PER CENT, DECIMAL, AND FRACTION EQUIVALENTS

$50\% = .50 = \frac{1}{2}$	$40\% = .40 = \frac{2}{5}$	$37\frac{1}{2}\% = .37\frac{1}{2} = \frac{3}{8}$
$25\% = .25 = \frac{1}{4}$	$60\% = .60 = \frac{3}{5}$	$62\frac{1}{2}\% = .62\frac{1}{2} = \frac{5}{8}$
$75\% = .75 = \frac{3}{4}$	$80\% = .80 = \frac{4}{5}$	$87\frac{1}{2}\% = .87\frac{1}{2} = \frac{7}{8}$
$33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}$	$16\frac{2}{3}\% = .16\frac{2}{3} = \frac{1}{6}$	$10\% = .10 = \frac{1}{10}$
$66\frac{2}{3}\% = .66\frac{2}{3} = \frac{2}{3}$	$83\frac{1}{3}\% = .83\frac{1}{3} = \frac{5}{6}$	$8\frac{1}{3}\% = .08\frac{1}{3} = \frac{1}{12}$
$20\% = .20 = \frac{1}{5}$	$12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$	$5\% = .05 = \frac{1}{20}$

Case I. How to Find a Per Cent of a Number

What is 25% of 48? 17% of 40?

$25\% = \frac{1}{4}$ $\overset{12}{\cancel{48}} \times \frac{1}{\cancel{4}} = 12$ Ans.

$17\% = .17$ $40 \times .17 = 6.8$ Ans.

Multiply the number by the per cent expressed as a fraction or a decimal, whichever is easier to work with.

Case II. How to Find What Per Cent One Number Is of Another

15 is what per cent of 200?

$15 = \frac{15}{200}$ of 200

$\frac{15}{200} = 200\overline{)15.00}$ $.07\frac{1}{2} = 7\frac{1}{2}\%$ Ans.

Proof: $.07\frac{1}{2} \times 200 = 15$

Write the two numbers as a fraction, using as the denominator the number of which the other is a part.

Change the fraction to a decimal. Change the decimal to a per cent.

Case III. How to Find a Number When a Per Cent of It Is Given

36 is 24% of what number?

24% of the number = 36

$1\% = \frac{1}{\cancel{24}_{2}} \times \overset{3}{\cancel{36}} = \frac{3}{2} = 1\frac{1}{2}$

$100\% = 1\frac{1}{2} \times 100 = 150$ Ans.

Or

$*.24_{\wedge}\overline{)36.00_{\wedge}}$ 150 Ans.

Proof: $.24 \times 150 = 36$

First find what 1% of the number equals. Then multiply the result by 100 to find 100%, or the number.

Or

Divide the given number by the per cent expressed as a fraction or a decimal, whichever is easier to use.

*Dividing 36 by .24 is the same as finding 1% of 36 and then 100%, because $36 \div .24$ or $\frac{24}{100} = 36 \times \frac{100}{24} = 150$.

Find:

	A	B	C
1.	$33\frac{1}{3}\%$ of 126	$12\frac{1}{2}\%$ of 96	25% of \$72
2.	75% of 84 bu.	$66\frac{2}{3}\%$ of 240	$62\frac{1}{2}\%$ of \$248
3.	14% of 3240	88% of 48	48% of 1750 lb.
4.	39% of 568	92% of 365	76% of 31.6

What per cent is:

5.	16 qt. of 80 qt.?	48 of 72?	125 of 500?
6.	480 A. of 600 A.?	96 of 400?	256 of 800?
7.	\$72 of \$3000?	318 of 1200?	584 of 1600?
8.	8.4 T. of 40 T.?	21.6 of 30?	.48 of 24?
9.	37.2 bu. of 248 bu.?	300 of 729?	400 of 520?

Find the number of which:

10.	12 = 25%	48 = $33\frac{1}{3}\%$	\$65 = $62\frac{1}{2}\%$
11.	36 = 6%	123 = 3%	315 ft. = 5%
12.	4.8 = 2%	7.6 = 4%	13.2 gal. = 12%
13.	216 = 18%	2303 = 49%	4824 lb. = 67%

14. In a spelling test of 60 words, Tom spelled 85% of the words correctly. How many words did he misspell in the test?

15. A woman left \$3600 to her son. This was 75% of all her money. How much money did she leave in all?

16. A city allowed \$28,500 for snow removal, but spent only \$25,080 for this purpose. What per cent of the allowance was not used?

17. A man pays \$120 a month for rent, which is 25% of his monthly salary. What is his yearly salary?

The whole of anything is 100% of it. 100% of a number equals the number. 500% of a number equals 5 times the number. What is 300% of a number? 450%? 125%?

Remember:

100% of a number equals the number.
Less than 100% of a number is less than the number.
More than 100% of a number is more than the number.

Find:

	A	B	C
1.	400% of 90	$112\frac{1}{2}\%$ of 72 T.	137.5% of \$500
2.	250% of 14 pk.	116% of 60 bu.	$187\frac{1}{2}\%$ of \$800
3.	175% of 64 mi.	308% of \$144	247.2% of \$775

4. Mr. Kent earned \$3000 last year. This was $133\frac{1}{3}\%$ of his earnings for the previous year. What were his earnings for the previous year?

PER CENTS LESS THAN 1%

The Clark Co. sets aside for advertising expenses $\frac{7}{8}$ of 1% of its yearly sales. How much does it set aside for this purpose from yearly sales amounting to \$48,000?

Method 1. 1% of \$48,000 = \$480. $\frac{7}{8}$ of 1% = $\frac{7}{\cancel{8}} \times \overset{60}{\cancel{\$480}} =$ \$420.

$\frac{7}{8}$ of 1% may also be written as $\frac{7}{8}\%$ or, decimally, as $.00\frac{7}{8}$, or .00875.

Method 2. $\frac{7}{8}\%$ = .00875. .00875 × \$48,000 = \$420.

Find the following, using whichever method is easier:

1. $\frac{3}{4}\%$ of 760
2. $\frac{1}{2}\%$ of 1450
3. $\frac{5}{6}\%$ of \$816
4. $\frac{2}{5}\%$ of 975 A.
5. $\frac{7}{8}\%$ of \$3024
6. $\frac{9}{10}\%$ of \$5985

FINDING THE PER CENT OF INCREASE OR DECREASE

This last year, the number of people employed in factories in our town increased from 12,730 to 13,840. Find the per cent of increase.

Think: Increase in factory workers = 13,840 − 12,730, or 1110.

1110 is what per cent of 12,730?

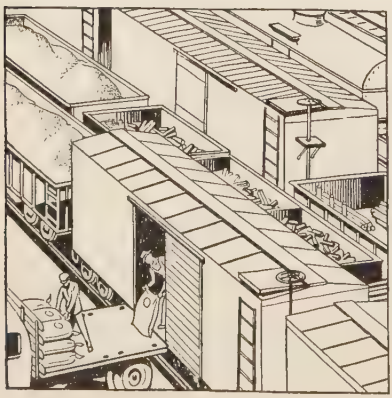
Per cent of increase $= \frac{1110}{12730} = 111 \div 1273 = .0871+$, or 8.7%.

Remember: To find the per cent of increase or decrease, divide the amount of the increase or decrease by the number that has been increased or decreased.

Find the per cent of increase or decrease that the second number shows as compared with the first number:

1. \$600, \$720	**4.** 840 bu., 945 bu.	**7.** \$2400, \$2592
2. \$800, \$680	**5.** 500 qt., 471 qt.	**8.** 6400, 5312
3. 1200, 1308	**6.** 7200 mi., 6984 mi.	**9.** \$880, \$818.40

10. Anna's weekly salary was increased from \$20 to \$22.50. What was the per cent of increase?



11. Last year, Mr. Hale's expense for truck repairs was \$2241; the year before, it was \$2700. Find the per cent of decrease.

12. In one week, 703,000 carloads of freight were shipped in our country. The next week, 727,000 carloads were shipped. Find the per cent of increase in carload shipments the second week.

Among all the sports, baseball stands first in America. The sports pages of the newspapers give much space to descriptions of the games and show the ranking of the teams and players. Girls as well as boys can follow the progress of their favorite teams and players if they know what the figures in the newspapers mean.

Joe DiMaggio was the leading batter in the American League in a recent year. Here is his record:

	Gms.	A.B.	R.	H.	H.R.	R.B.I.	S.B.	P.C.
DiMaggio	120	462	107	176	30	126	3	.381

Boys, tell the girls what the above abbreviations mean.

A player's batting average is found by dividing the number of hits he made by the number of times he was at bat.

Batting averages are usually written as three-place decimals but are called per cents. If a player's batting average is .381, we say, for short, that it is 381.

How to Find a Player's Batting Average

A leading batter in the National League made 167 hits in 489 times at bat. Find his batting average.

$489\overline{)167.0000} = .3415$ = .342 (in baseball, usually read *three forty-two*)

Divide the number of hits by the number of times at bat. Carry your answer to four decimal places, and round it to the nearest thousandth.

The player's batting average was .342.

1. At the right are listed the numbers of hits and times at bat of eight leading major league batters in a recent season. Find the batting averages.

2. A player's batting average was .344. His number of times at bat was 500. How many hits did he make?

Batter	At Bat	Hits	P.C.
A	158	55	?
B	489	167	?
C	531	179	?
D	640	209	?
E	541	175	?
F	590	190	?
G	608	195	?
H	427	134	?

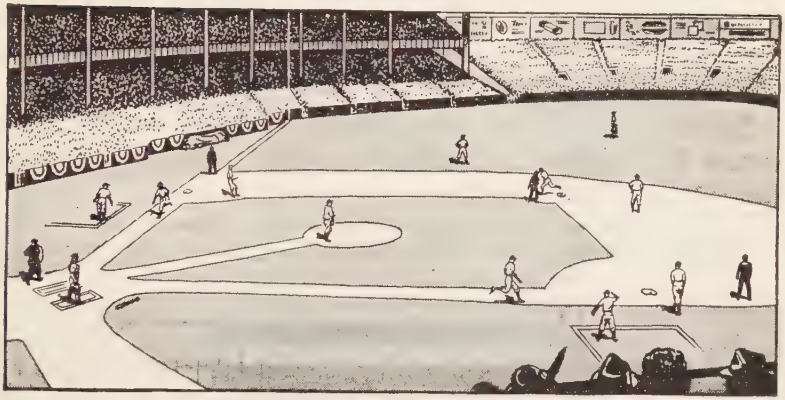
BASEBALL TEAM STANDINGS

In a baseball league, each team is ranked according to the per cent of games it has won. This per cent is found by dividing the number of games won by the number of games played, and is written as a three-place decimal. Like a batting average, this decimal is called a per cent.

At the end of a recent playing season, the Cincinnati Reds of the National League had won 97 games and lost 57. What per cent did the team make? $97 + 57 = 154$ games played. $97 \div 154 = .6298 = .630$, or 63%.

1. The standings of the National League teams and the American League teams at the end of a recent season are given below. Find the missing per cents. Is each team ranked correctly?

National League				American League			
Team	Won	Lost	P.C.	Team	Won	Lost	P.C.
Cincinnati	97	57	.630	New York	106	45	?
St. Louis	92	61	?	Boston	89	62	?
Brooklyn	84	69	?	Cleveland	87	67	?
Chicago	84	70	?	Chicago	85	69	?
New York	77	74	?	Detroit	81	73	?
Pittsburgh	68	85	?	Washington	65	87	?
Boston	63	88	?	Philadelphia	55	97	?
Philadelphia	46	106	?	St. Louis	43	111	?



A home run with two on the bases.

1. The first world champion series of baseball games was played in 1903 between the Boston American League team and the Pittsburgh National League team. The Boston team won 5 games; and the Pittsburgh team, 3. What per cent of the games played did each team win?

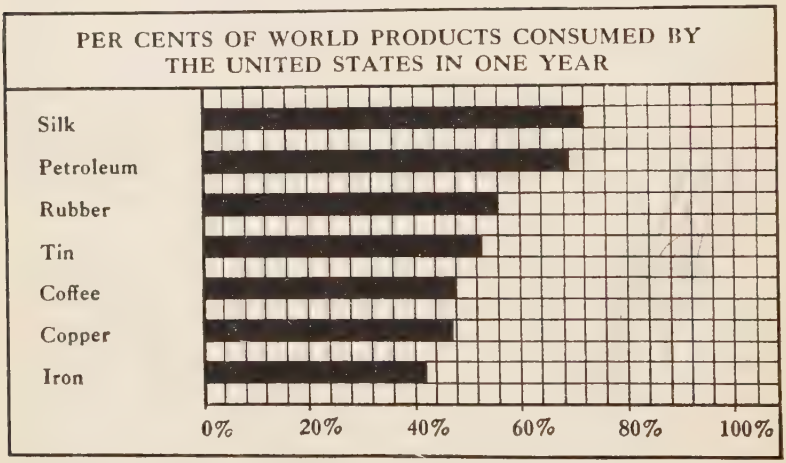
2. In the next World Series played, the New York National League team won 4 games and the Philadelphia American League team won 1 game. What per cent of the games did each team win?

3. In a four-game series between the New York American League team and the Chicago National League team, the New York team made 37 hits in 135 times at bat, and the Chicago team made 33 hits in 136 times at bat. What was the batting average of each team?

4. The paid attendance for the four games between the New York Americans and the Chicago Nationals was 200,832. The total receipts were $851,166. Find the average attendance and the average receipts for each game.

Per cents are easy to compare because they are based on the same denominator, 100. For this reason, per cents are often used to compare numbers.

In the graph below, the quantity of each product consumed by the United States is expressed as a per cent of the total world production.



1. For what does 100% in the graph stand?

2. Read the graph, giving as accurately as possible the per cent of the production of each product which we used.

3. Show the above facts by a line graph.

4. The United States has about 50% of the world's telephones, 40% of the world's railroad mileage, and 70% of the world's automobiles. Show these facts by pictographs.

5. There are 1800 pupils in the Hamilton School. Of these, 180 pupils are in the kindergarten; 648, in the primary grades; 558, in the grammar grades; and the rest, in the high-school grades. Show these facts as per cents in a bar graph.

Work the examples in column A. If you fail on any example, do the examples opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill		C Review Pages
1. Finding a Per Cent of a Number			
$66\frac{2}{3}\%$ of 1296 = ?	$87\frac{1}{2}\%$ of 456 = ?	80% of 975 = ?	325
250% of 16 = ?	125% of 60 = ?	350% of 48 = ?	
$\frac{3}{5}\%$ of 3500 = ?	$\frac{5}{6}\%$ of 5400 = ?	$\frac{3}{8}\%$ of 7200 = ?	
23% of 458 = ?	47% of 625 = ?	89% of 1260 = ?	
2. Finding What Per Cent One Number Is of Another			
72 = ?% of 96	48 = ?% of 60	45 = ?% of 75	325
144 = ?% of 48	56 = ?% of 24	175 = ?% of 50	
97 = ?% of 400	73 = ?% of 500	147 = ?% of 600	
.9 = ?% of 150	1.5 = ?% of 200	.1 = ?% of 16	
3. Finding a Number When a Per Cent of It Is Given			
42 = 40% of ?	68 = $33\frac{1}{3}\%$ of ?	36 = 75% of ?	325
7.6 = 4% of ?	5.4 = 3% of ?	6.4 = 16% of ?	
84 = 350% of ?	85 = 250% of ?	45 = 125% of ?	
8 = $\frac{2}{3}\%$ of ?	28 = $\frac{4}{5}\%$ of ?	35 = $\frac{5}{6}\%$ of ?	
4. Finding the Per Cent of Increase or Decrease			
Marked \$42.30 Sold at \$25.38 Decrease = ?%	Marked \$700 Sold at \$560 Decrease = ?%	Marked \$18.75 Sold at \$17.25 Decrease = ?%	328
July sales, \$625 Aug. sales, \$900 Increase = ?%	Feb. sales, \$840 Mar. sales, \$960 Increase = ?%	Oct. sales, \$120 Nov. sales, \$160 Increase = ?%	

Make a problem from each of the following by framing a suitable question. If possible, build more than one question for each. Work the problems you make.

1. Last year, Mr. Harper raised 3100 bu. of corn. This year, he raised 4960 bu.

2. In a shipment of 600 baskets of tomatoes, 18 baskets were found to be below grade.

3. In Reed City, 92% of the 25,850 school children were vaccinated.

4. In 6 history tests, Fred received the following marks: 71%, 92%, 100%, 64%, 88%, and 70%.

5. Mr. Dodd sold 120 acres of farm land. This was 66% of his farm.

6. Bill worked 52 da. during vacation and earned $1.50 a day. He saved 45% of his earnings.

7. Mr. Barker sold $1250 worth of merchandise in September. His October sales were 28% more than his September sales.

8. In a recent year, the world crop of wheat amounted to 4,598,790,000 bu. In the same year, the wheat crop of the United States was 626,461,000 bu.

9. As a result of safety campaigns, the number of deaths from traffic accidents in the United States for a recent year was 7640 less than for the preceding year. The number of traffic deaths the preceding year was 39,643.

Following the winter holidays, Mr. Faber offered his entire stock of fur coats at 20% to 35% reduction from the usual prices. The peak of the buying season had passed, and he did not want his money tied up in stock which he would have to carry over to another winter. He also wished to save the expense of storage and insurance. Then, too, styles in coats might change, leaving him with out-of-date merchandise.

In order to get back money invested, to increase sales, or to make room for new stock, merchants often offer goods at reductions from the usual prices.

The usual price of an article is called the *regular price*.

The amount allowed off the regular price is called the *discount*.

The per cent of discount is called the *rate of discount*.

The price at which an article is sold after the discount has been deducted from the regular price is called the *net price*.

Find the discounts and the net prices when the regular prices and rates of discount are as follows:

1. \$720, $33\frac{1}{3}\%$	**5.** \$2.98, 5%	**9.** \$115, 15%
2. \$275, 20%	**6.** \$10.50, $12\frac{1}{2}\%$	**10.** \$384.20, 40%
3. \$40, 12%	**7.** \$560, $7\frac{1}{2}\%$	**11.** \$630.25, $16\frac{2}{3}\%$
4. \$300, 8%	**8.** \$897, 25%	**12.** \$942, 35%

A desk regularly priced at $42.50 is sold at a discount of 30%. Find the net price.

A short way to find the net price: **Regular price = 100%. 100% − 30% discount = 70%. Net price = 70% of regular price. .70 × $42.50 = $29.75, net price.**

Find the net prices of articles marked as follows, less the rates of discount given:

13. Typewriter, $105, 20%
14. Tractor, $582.50, 50%
15. Mattress, $39.50, 25%
16. Piano, $435, $33\frac{1}{3}\%$

FINDING THE PER CENT OF DISCOUNT

How to Find the Per Cent of Discount

A chair priced at $20 was sold for $15. What was the per cent of discount?

$20 − $15 = $5 discount — Find the discount.

$5 = $\frac{5}{20}$ or $\frac{1}{4}$ of $20 — Find what fraction the discount is of the regular price.

$\frac{1}{4}$ = .25 = 25% discount — Change the fraction to a per cent.

Or

$5 ÷ $20 = .25 — To find the per cent of discount, divide the discount by the regular price.

.25 = 25% discount

Find the rate of discount in each of the following:

	Regular Price	Discount		Regular Price	Net Price
1.	$42	$6.30	5.	$3350	$3082
2.	$120	$30	6.	$1725	$1621.50
3.	$186	$31	7.	$2628	$1752
4.	$724	$90.50	8.	$5400	$4500

How to Find the Regular Price

The sale price of a pair of skates sold at a discount of 20% was $8. What was the regular price?

Regular price = 100%

100% − 20% discount = 80%

$8 sale price = 80% of regular price

1% of regular price = $\frac{1}{80} \times \$8.00 = \$.10$

100% of regular price = $\$.10 \times 100 = \$10$

Or

100% − 20% = 80%

80% = .80

*$8 ÷ .80 = $10 regular price

Find what per cent of the regular price the sale price equals.

Then find what 1% of the regular price equals, and multiply this amount by 100 to find 100%, or the regular price.

Or

To find the regular price, divide the net price by the per cent that represents the net price.

*Dividing $8 by .80 is the same as finding 1% of $8 and then 100%, because $8 ÷ .80 or $\frac{80}{100} = \$8 \times \frac{100}{80} = \$10$.

Find the regular price in each of the following:

	Net Price	Discount		Net Price	Discount
1.	$21	25%	4.	$17.50	30%
2.	$68	20%	5.	$594	$33\frac{1}{3}\%$
3.	$7.50	$16\frac{2}{3}\%$	6.	$160.14	15%

7. John bought a radio for $72, which was $33\frac{1}{3}\%$ less than the regular price. What was the regular price?

8. The selling price of a bicycle sold at a discount of 20% was $32. Find the regular price and the discount.

Merchants who sell directly to the people who use their goods are known as **retail merchants.** Merchants who sell to retail merchants are called **wholesale merchants.** Wholesale merchants bring out catalogues containing the **list prices** of their goods. From these list prices, retail merchants are allowed **trade discounts.**

In addition to the regular trade discount, a wholesale dealer may give further discounts if an order is large or if the cost of goods has gone down. In such cases, the first discount is taken off the list price, and additional discounts are taken from the successive remainders. Such discounts are called **successive discounts.**

How to Find the Net Price When Successive Discounts Are Given

The Crawford Company bought plumbing supplies listed at \$1640, less successive discounts of 25%, 10%, and 2%. Find the net price of the goods.

List price	\$1640
25% discount	410 ($\frac{1}{4} \times$ \$1640)
	\$1230
10% discount	123 ($\frac{1}{10} \times$ \$1230)
	\$1107.00
2% discount	22.14 (.02 $\times$ \$1107)
	\$1084.86 net price

Find the amount of the first discount, and subtract it from the list price.

Find the second discount, figuring it on the amount remaining after the first discount was deducted, and subtract the second discount from that amount.

Find any additional discounts in the same way.

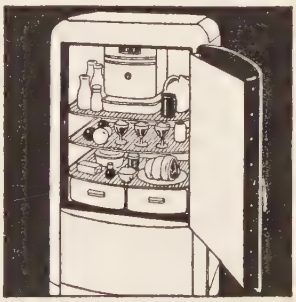
1. Are a single discount of 37% and successive discounts of 25%, 10%, and 2% the same? Prove your answer.

2. Work the example in the box, reversing the order of the discounts. Is the result the same?

Find the net prices of the following. Figure each discount to the nearest cent.

	List Price	Discounts		List Price	Discounts
1.	\$1200	30%, 10%	5.	\$840	20%, 10%, 2%
2.	\$960	25%, 5%	6.	\$2000	25%, $16\frac{2}{3}$%, 5%
3.	\$188.40	$12\frac{1}{2}$%, 3%	7.	\$780	$33\frac{1}{3}$%, 10%, 5%
4.	\$824	30%, 5%	8.	\$1680	25%, 10%, 2%

9. A dealer buys refrigerators in large quantities at list prices less successive discounts of $33\frac{1}{3}$% and 5%. How much does he pay for 3 doz. refrigerators listed at \$98?



10. A stove listed at \$125 was sold at wholesale with successive discounts of 25%, 10%, and 2%. Find the net price of the stove.

11. Mr. Hart can buy a certain radio from one dealer for \$80 less a single discount of 3%, or from another dealer for \$100 less discounts of 15% and 10%. Which dealer offers the lower net price? How much lower?

12. A dealer wishes to order a gas heater. The list price is \$168. He can order from one firm at successive discounts of 15%, 15%, and 5%, or from another firm at a single discount of 30%. Which price is better? How much better?

13. The list price of a set of tools is \$160. One dealer offers discounts of 20% and 5%; another dealer offers a single discount of 25%; and a third dealer, discounts of 5% and 20%. Find the net price in each case. Which is the best offer? What is the saving between the highest and the lowest offer?

An **invoice** is an itemized bill sent by a dealer to a customer at the time the order is shipped. Goods should be checked against the invoice as soon as received.

Invoice #2036

ALBERT BAKER & CO.
NEW HAVEN, CONNECTICUT

April 1, 19__

Sold to J. B. Davis & Son
Trenton, New Jersey

Terms: 2%,10 da.; net, 30

15	Watches	@ $30	450	00		
12	Cameo rings	@ $7.50	90	00		
1	Diamond ring		62	50	602	50
		Discounts 20%, 5%			144	60
					457	90

The terms **2%, 10 da.; net, 30** mean that if payment is made within 10 days, 2% **cash discount** will be allowed from the net amount of the invoice. Otherwise, the full net amount is due at the end of 30 days.

Notice that the invoice gives the **quantity** of each item purchased; the name of the **item;** the **list price;** and the **extension,** or total for each item. What other information does it give?

1. Check the extensions in the above invoice and the total of the three items at list price.

2. Check to see if the amount of the discounts is correct.

3. Davis & Son paid the above invoice within 10 days. Find the amount of the invoice less the cash discount. Then copy the invoice, showing "2% cash discount" deducted. Receipt the invoice by writing the words **Received Payment** followed by the date April 10, 19—, the name of the firm, and a cashier's initials.

Some people earn their living by doing business for others and charging for their services a per cent of the amount of business done. Such people are known as **agents.**

The money that is paid to an agent for his services in buying or selling goods or property, or in doing business for another, is called *commission.* The per cent of commission is called the *rate of commission.*

An agent who buys or sells for others such things as fruits and garden products is called a *commission merchant.*

When an agent buys goods for another, he figures his commission on the purchase price; that is, the amount he pays for the goods. He adds to the purchase price his commission and any charges for freight or other expenses he has had to pay. This amount is the **total cost** to the person for whom the agent purchased the goods.

When an agent sells goods for another, he figures his commission on the selling price; that is, the amount he receives for the goods. He deducts from the selling price his commission and any charges he has had to pay. The difference is the **net proceeds,** or the amount the agent gives to the person whose goods he sold.

1. Mr. Adams bought through an agent 500 bu. of wheat at 80¢ a bushel. The agent charged $1\frac{1}{2}\%$ commission, and the transportation charges were 5¢ a bushel. Find the total cost of the wheat to Mr. Adams.

2. An agent sold some cattle for Mr. Darrow for $4960. He charged 5% commission and freight charges of $96.80. Find his commission and the net proceeds for Mr. Darrow.

3. An insurance agent received $3125 commission on insurance payments amounting to $12,500. What was his rate of commission? ($3125 is what per cent of $12,500?)

Find the missing numbers in the following:

	Sales	Rate of Com.	Com.	Net Proceeds	Pur. Price	Charges	Total Cost
1.	$650	5%	?	?		none	
2.		3%	?		$840	$15	?
3.	$250	?	$15	?		none	
4.		2%	?		$175	$2.50	?
5.	$2400	4%	?	$2206		?	
6.		$1\frac{1}{2}$%	?		$2062.50	$262.50	?
7.	$8960	?	$761.60	?		none	

8. A sales clerk is paid a salary of $5 a day. He receives in addition a commission of 2% of his daily sales above $50. On Wednesday his sales amounted to $97.50. What were his total earnings for that day?

9. A commission merchant received 60 cases of eggs, each case containing 30 dozen. He sold the eggs at 22¢ a dozen and charged $15.84 commission. What rate of commission did he charge?

10. A real-estate agent sold a house for Mr. Stone for $6500. The agent charged 5% commission on the first $1000 and $2\frac{1}{2}$% on the remainder. (a) How much did the agent receive for selling the property? (b) Find the net proceeds Mr. Stone received from the sale of the property.

11. Mr. Hall bought 550 bu. of peaches through a commission merchant. The peaches cost $1.05 a bushel, and freight and cartage expenses were $85. The agent's commission was 4%. How much was the commission? What was the total cost of the peaches to Mr. Hall?

Work the examples in column A. If you fail on any example, do the examples opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review
1. Finding the Discount and the Net Price		
Reg. Pr. \$1824, Disc. 3%	Reg. Pr. \$1260, Disc. $33\frac{1}{3}\%$; Reg. Pr. \$888, Disc. 4%	335
2. Finding the Per Cent of Discount		
Reg. Pr. \$17.84, Net Pr. \$15.70	Reg. Pr. \$372, Net Pr. \$341; Reg. Pr. \$776, Net Pr. \$679	336
3. Finding the Regular Price		
Net Pr. \$645, Disc. $16\frac{2}{3}\%$ Net Pr. \$441.37, Disc. $4\frac{1}{2}\%$	Net Pr. \$927, Disc. 25%; Net Pr. \$836, Disc. 20% Net Pr. \$501.27, Disc. $18\frac{1}{2}\%$; Net Pr. \$303.89, Disc. $22\frac{1}{2}\%$	337
4. Finding the Net Price When Successive Discounts Are Given		
List Price \$560, Discounts 25%, 15% List Price \$896, Discounts 30%, 5%	List Price \$750, Discounts $33\frac{1}{3}\%$, 10%; List Price \$180, Discounts 30%, 20% List Price \$620, Discounts 20%, 2%; List Price \$918, Discounts $16\frac{2}{3}\%$, 3%	338
5. Finding the Total Cost		
Pur. Pr. \$256 Com. 4% Exp. \$18	Pur. Pr. \$504 Com. 5% Exp. \$12; Pur. Pr. \$346 Com. 3% Exp. \$17	341
6. Finding the Commission and the Net Proceeds		
Sales \$456.50, Com. 5% Charges \$5.10	Sales \$309.40, Com. 4% Charges \$14.50; Sales \$88.50, Com. 3% Charges \$3.16	341

Mr. Walsh has an electrical appliance store. One day, his son Paul was helping him to mark price tags for radios. "This radio cost $62.50," said his father. "Make the selling price $93.75."

"That means you will make $31.25 **gain,** or **profit,** on the radio, doesn't it?" asked Paul.

"Oh, no," his father replied. "Only part of that is profit. The rest pays such expenses as rent, heat, light, wages—including my own salary, and other expenses. If I don't sell the radio within a reasonable length of time, I may have to mark the price lower, and my profit will be smaller. If I have to carry the radio too long, I may have to sell it at a loss. In order to avoid losses, it is better to take a smaller profit and use the money in buying new goods that will sell quickly."

The cost of merchandise, including transportation charges and any expense for direct repairs, is known as the *first cost.*

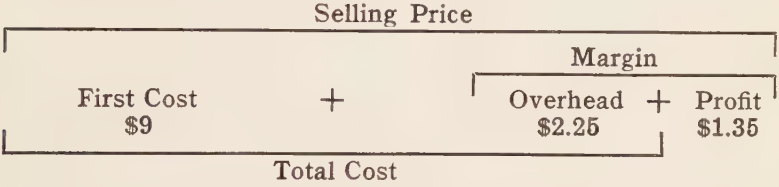
All store expenses except the first cost of the goods are called *operating expenses* or *overhead.*

The cost of overhead and an allowance for profit must be added to the first cost of goods in figuring the **selling price** if business is to be done at a profit.

The first cost plus the overhead is called the *total cost.*

The overhead plus the profit is called the *margin.* In other words, the margin is the difference between the first cost and the selling price.

1. Study the following diagram which shows the first cost of an article, the overhead, and the profit. Find the total cost, the margin, and the selling price.



Any successful business must make a profit in order to have a reserve on hand to meet unexpected losses, to expand the business, and, if possible, to pay a return in money to those who have invested money in the business.

Profits may be increased by a rapid **turnover;** that is, the quick sale and replacement of goods. A rapid turnover allows a merchant to buy and sell a greater volume of goods each year than if his turnover were slow. He can thus afford to sell at a smaller profit because his many little profits make a large profit in the course of a year.

SOME THINGS TO DISCUSS

1. Rents are higher in the business district of a city than in the surrounding districts. Yet the prices in a large downtown store may be lower than those charged for the same articles in an outlying store. Explain.

2. Name some of the things which a store can do to attract customers.

3. Is an article bought at a low price always a bargain? Explain.

4. Many large grocery stores now have "self service." What is meant by this? What are some of the advantages of self service to the storekeeper? To the customer?

Some merchants figure profit and loss as a per cent of the first cost of goods. When profit or loss is based on the first cost, the first cost equals 100%.

The word **cost** is used hereafter to mean **first cost.**

How to Find the Profit and the Selling Price

Mr. Colt paid $600 for a tractor and spent $50 for repairing it. He sold the tractor at a profit of 6% of the cost. Find his profit and the selling price, based on the cost.

$600 + $50 = $650 cost	Find the cost.
6% = .06	Find the profit by multiplying the cost by the per cent of profit.
$650 × .06 = $39 profit	
$650 + $39 = $689 S.P.	Add the cost and the profit to find the selling price.

How to Find the Per Cent of Profit

A dealer bought bicycles at $24 each and sold them at $29. His overhead was $12\frac{1}{2}\%$ of the cost. Find the profit and the per cent of profit, based on the cost.

$12\frac{1}{2}\% = \frac{1}{8}$	Find the overhead by multiplying the cost by the per cent of overhead.
$\$24 \times \frac{1}{8} = \$3$ overhead	Add the overhead and the cost to find the total cost.
$24 + $3 = $27 total cost	Subtract the total cost from the selling price to find the profit.
$29 − $27 = $2 profit	
$2 ÷ $24 = $.08\frac{1}{3} = 8\frac{1}{3}\%$ profit	

Divide the profit by the cost to find the per cent, or rate, of profit.

To find the *loss*, multiply the cost by the per cent of loss. *Subtract* the loss from the total cost to find the selling price. To find the per cent of loss, divide the loss by the cost.

Find the missing numbers in the following, basing profit, loss, and overhead on the cost:

	Cost	% Profit	% Loss	Over-head	Selling Price	Profit or Loss
1.	$210	20%		$52.50	?	?
2.	$36	?		22%	$49.32	?
3.	$176		$12\frac{1}{2}$%	$24.64	?	?
4.	$14.60		?	10%	$15.33	?

PROFIT AND LOSS BASED ON SELLING PRICE

Many business men prefer to figure profit or loss as a per cent of the selling price just as they do commissions and discounts. In this case, the selling price equals 100%.

To find the profit or the loss based on the selling price, multiply the selling price by the per cent of profit or loss expressed as a fraction or a decimal.

To find the per cent of profit or loss based on the selling price, divide the profit or loss by the selling price and express the quotient as a per cent.

Find the missing numbers in the following, basing profit, loss, and overhead on the selling price:

	Cost	Total Cost	% Profit	% Loss	Over-head	S.P.	Profit or Loss
1.				?	none	$392	$49
2.	$17.60	?	?		7%	$20	?
3.	$62	?		?	$4.50	$65	?
4.	?	?	15%		20%	$12	?
5.	$83.60	?	?		14%	$110	?

How to Figure the Selling Price, Overhead and Profit Based on the Cost

At what price must Mr. Dunn sell a set of golf clubs costing $12 in order to allow 25% for overhead and 20% for profit, based on the cost?

$25\% + 20\% = 45\%$ margin

$\$12 \times .45 = \$5.40$ margin

$\$12 + \$5.40 = \$17.40$ S.P.

Add the per cents of overhead and profit to find the per cent of margin.

Multiply the cost by the per cent of margin to find the margin.

Add the cost and the margin to find the selling price.

How to Figure the Selling Price, Overhead and Profit Based on the Selling Price

Mr. Ogden buys clocks at $14 each. At what price must he sell them in order to allow 15% for overhead and 15% for profit, based on the selling price?

$15\% + 15\% = 30\%$ margin

$100\% - 30\% = 70\%$

$\$14$ cost $= 70\%$ of S.P.

Find the per cent of margin, and subtract it from 100% to find the per cent that represents the cost.

1% of S.P. $= \frac{1}{70} \times \$14.00 = \$.20$

$100\% = \$.20 \times 100 = \$20$ S.P.

Or

$70\% = .70$

$.70\overline{)\$14.00}$

Find what 1% of the selling price equals, and multiply this amount by 100 to find 100%, or the selling price.

Or

$\$20$ S.P.

To find the selling price, divide the cost by the per cent that represents the cost.

Find the selling price in each of the following, basing overhead and profit on the cost:

	Cost	Overhead	Profit		Cost	Overhead	Profit
1.	\$120	none	20%	5.	\$40	24%	$16\frac{1}{2}\%$
2.	\$900	20%	$16\frac{2}{3}\%$	6.	\$7.50	22%	20%
3.	\$420	25%	15%	7.	\$64.50	25%	18%
4.	\$84	$12\frac{1}{2}\%$	$7\frac{1}{2}\%$	8.	\$10.98	20%	$16\frac{2}{3}\%$

Find the selling price in each of the following, basing overhead and profit on the selling price:

	Cost	Overhead	Profit		Cost	Overhead	Profit
9.	\$1666	none	15%	13.	\$6.75	18%	12%
10.	\$1190	22%	10%	14.	\$65.45	20%	12%
11.	\$476	12%	8%	15.	\$52.50	$12\frac{1}{2}\%$	$7\frac{1}{2}\%$
12.	\$207	25%	15%	16.	\$.975	25%	9.2%

17. At what price should fishing rods costing \$4.20 each be sold in order to allow 24% for overhead and 20% for profit, based on the cost?

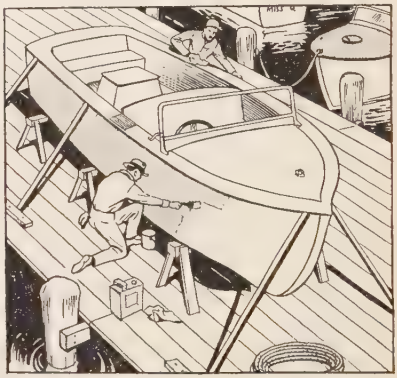
18. Dress goods costing 48¢ a yard must be sold at what price to allow 18% for overhead and 15% for profit, based on the cost?

19. Mr. Henry bought a lot for \$800 and built a house on it at a cost of \$6400. At what price must he sell the property in order to allow 10% for selling expenses and 15% for profit, based on the selling price?

20. Mr. Carter bought lamps listed at \$4.50 each, less discounts of 20% and 10%. At what price should he sell them in order to allow 15% for overhead and 20% for profit, based (a) on the cost; (b) on the selling price?

1. Mr. Bell bought a house for \$8500 and sold it for \$8330. What was his per cent of loss, based on the cost?

2. Mr. Carr sells stoves. What does he charge for a gas range costing him \$56, if he allows a margin of 30%, based on the selling price?



3. A dealer spent \$85 for repairs to a boat that had cost him \$400 second-hand. He then sold the boat at a profit of 12%, based on the cost. Find his profit and the selling price.

4. Mr. Kean, who sells house furnishings, allows 20% for overhead and 10% for profit, based on the cost. At what price will he sell hooked rugs that cost him \$12.50 each?

5. A farm cost \$7500. An amount equal to 20% of the cost was spent for improvements. The farm was then sold for \$12,000. What was the profit? The per cent of profit, based on the cost?

6. A kitchen cabinet that cost \$11.60 was sold at a price that included an allowance of 15% for overhead and $12\frac{1}{2}$% for profit, based on the selling price. What was the selling price?

7. A case of a certain cereal costs \$2.40 delivered. It contains 20 packages. At what price per package must the cereal be sold to allow 15% for overhead and 10% for profit, based on the cost?

Work the examples in column A. If you fail on any example, do the examples opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test			B Drill				C Review
1. Finding Profit or Loss and Selling Price, Based on Cost							
Cost		Profit	Cost	Profit	Cost	Loss	
$65.85		20%	$88	25%	$72	$16\frac{2}{3}\%$	346
2. Finding the Per Cent of Profit or Loss, Based on Cost							
Cost	Overhead	S. P.	Cost	Overhead	S. P.		
$400	$185	$599	$520	$94.75	$653.75		346
$82.80	$16\frac{2}{3}\%$	$103.80	$30	$33\frac{1}{3}\%$	$39.10		
3. Finding the Profit or Loss, Based on Selling Price							
S. P.		Profit	S. P.	Loss	S. P.	Profit	
$260		15%	$470	11%	$168	$12\frac{1}{2}\%$	347
4. Finding the Per Cent of Profit or Loss, Based on Selling Price							
Cost	Overhead	S. P.	Cost	Overhead	S. P.		
$11.36	$3.20	$16.00	$19.10	$4.70	$28.00		347
$14.66	12%	$17.45	$29.61	20%	$32.90		
5. Figuring the Selling Price, Overhead and Profit Based on Cost							
Cost	Overhead	Profit	Cost	Overhead	Profit		
$162	19%	13%	$285	17%	9%		348
$23.50	15%	3%	$14.95	12%	4%		
6. Figuring the Selling Price, Overhead and Profit Based on Selling Price							
Cost	Overhead	Profit	Cost	Overhead	Profit		
$15.96	None	25%	$7.80	None	20%		348
$372	25%	15%	$520	15%	20%		

There is something missing from each of the following problems. Write each problem, supplying the missing fact. Work the problems.

1. Mr. Burns saves 10% of his monthly salary. How much does he save each month?

2. Our basketball team won 12 of its games last season. What per cent of its games did the team win?

3. John paid 40% of the price of a new bicycle as a first payment. How much does he still owe?

4. This winter, eggs cost 5¢ more a dozen than they did last winter. What is the per cent of increase?

5. At a sale, Carl bought a camera at a discount of 25% from the regular price. What was the regular price?

6. In addition to a salary, Mr. Martin receives a 3% commission on his sales over $40,000. Last year, his sales amounted to $76,400. What were his total earnings for the year?

7. A wholesale dealer sold a canoe less successive discounts of 15%, 10%, and 2%. What was the net price of the canoe?

8. After spending $450 for improvements, Mr. Hall sold a house for $6500. What was his per cent of profit, based on the selling price?

9. Mr. Cox bases his per cents of overhead and profit on the cost. At what price will he mark leather jackets which cost him $2.16?

Find the answers to these examples:

1. 4074 + 762 + 5379
2. 120583 less 23804
3. \$3799.95 ÷ 385
4. 476 times 1790
5. 64.7 minus 19.377
6. 84.02 multiplied by .093
7. 32.5 + 86.83 + 4.67
8. 23.495 divided by .074
9. Find the average of these numbers: 16, 20, 15, 13, 19, 22.

Find the answers. Watch the signs.

10. $\frac{3}{4} + \frac{7}{20} + \frac{5}{8}$
11. $88 \times 4\frac{5}{16}$
12. $1\frac{2}{3} + 4\frac{5}{6} + 3\frac{2}{15}$
13. $4\frac{3}{8} \times \frac{4}{9} \times 7\frac{1}{2}$
14. $14\frac{2}{3} \div 3\frac{1}{7}$
15. $36 - 17\frac{7}{24}$
16. $\frac{7}{9} \times \frac{15}{16}$
17. $16\frac{1}{4} - 3\frac{3}{8}$
18. $\frac{7}{24} \div \frac{9}{16}$
19. $82\frac{1}{7} - 35\frac{2}{3}$
20. $23\frac{3}{7} \times 10\frac{1}{2}$
21. $396\frac{2}{3} \div 8$

22. Reduce to simplest form: $\frac{39}{9}$, $\frac{62}{12}$, $\frac{72}{120}$, $\frac{77}{165}$, $\frac{80}{18}$.

23. Change to decimals: $\frac{12}{25}$, $\frac{7}{9}$, $\frac{3}{16}$, $\frac{7}{24}$, $\frac{5}{12}$.

24. Change to mixed numbers or fractions in lowest terms:

37.8 .66 72% $63\frac{1}{3}\%$ .425 148% 5.4%

25. Express the following ratios (a) as fractions in lowest terms; (b) as decimals; (c) as per cents.

12 to 30 32 to 50 22 to 4 120 to 500

24	576	19.2	480	\$.72	14.16	\$9.12	762.4

Use short cuts to work the following:

26. Multiply each number in the box by 5, 25, 50, 125.

27. Divide each number in the box by $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$.

Show that you know the meaning of each of the following terms by either (1) briefly defining it; or (2) using it correctly in a sentence:

1. List price
2. Per cent
3. Margin
4. Total cost
5. Quotient
6. Wholesale merchant
7. Trade discounts
8. Rate of commission
9. Terms of an invoice
10. Retail merchant
11. Net amount
12. Commission
13. First cost
14. Net price
15. Numerator

Match the terms in Column A with the descriptions in Column B:

Column A	Column B
16. Discount—	agents who sell fruits and garden products for others for a per cent of the selling price
17. Agents—	the amount of the sale after the commission and other charges have been deducted
18. Turnover—	all store expenses except the first cost of goods
19. Invoice—	the usual price of an article
20. Successive discounts—	a discount allowed for prompt payment of a bill
21. Commission merchants—	the rapidity with which goods are sold and replaced
22. Cash discount—	several discounts allowed on the same item or bill
23. Net proceeds—	an itemized bill sent by a dealer at the time the goods are shipped
24. Overhead—	the amount allowed off the regular price
25. Regular price—	people who buy or sell for others for a commission

1. Round .6384 to the nearest hundredth, and change the rounded decimal to a per cent.

2. In dividing 7368 by 395, what trial divisor should you use? Find the quotient to two decimal places.

3. Which of the following are equivalents: $\frac{5}{8}$, .0625, $66\frac{2}{3}\%$, .625, $62\frac{1}{2}\%$, $\frac{15}{24}$?

Work these examples:

4. 438 rd. = ? ft.

5. $\frac{5}{16}$ = ? per cent

6. $5\overline{)12 \text{ bu. } 2 \text{ pk.}}$

7. $6\frac{2}{3} \times 3\frac{7}{12} \times 4\frac{1}{2}$

8. Multiply 6748 by 25, using the short method.

Give the missing words:

9. Selling price = ___ + ___ + ___.

10. Loss = ___ − ___.

11. To find the regular price, divide the ___ by the ___.

12. What is the difference between $\frac{3}{4}$ of 12 and $\frac{3}{4}\%$ of 12?

13. How do you find the selling price when the cost is given and the per cents of overhead and profit are based on the selling price?

14. The list price of an article is $20. Which will give the cheaper net price: successive discounts of 20% and 10%, or a single discount of 30%? How much cheaper?

15. Would you rather make a profit of 25% based on the selling price or 25% based on the cost? Why?

16. If you know the price of $2\frac{1}{2}$ doz. bananas, how can you figure the price of 4 doz.?

WASHINGTON, Feb. 16—The Association of American Railroads reported today that 579,918 cars of freight were loaded during the week ending last Saturday, an increase of 36,927 cars over the figure for the same period a year ago.

1. Read this newspaper item and find the number of freight cars loaded during the same period a year ago. Round the number to the nearest thousand.

2. The pineapple industry in Hawaii gives continuous employment to 14,000 workers. During the harvest season, this number increases to about 35,000. About what is the per cent of increase during the harvest season?

3. A boy collects old newspapers, paying 15¢ a hundred pounds, and sells them to a furniture factory for $9 a ton. In one month, he sold 4000 lb. What was his profit for that month?

WASHINGTON, Feb. 20—The corn crop forecast today by the Federal Crop Reporting Board was about 7% less than last year's harvest of 2,644,995,000 bu.

4. According to this newspaper item, about what crop of corn is expected? Round your answer to the nearest million.

5. Mr. Tisdale bought $1235 worth of trees and shrubs through an agent. The transportation charges were $56, and the agent's commission was $5\frac{1}{2}\%$. Find the total cost of the plants to Mr. Tisdale.

6. Of the 5762 people who attended a school football game, 3542 were pupils. What per cent were not pupils?

7. Last year, the total sales of the Wiltshire Company were $1,268,000. The operating expenses were $278,960 and the profit was $95,100. Show the per cents of overhead, profit, and cost in a bar graph, letting the sales equal 100%.

Try to work these problems without using pencil or paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. Express one-eighth of 1% as a decimal.

2. What is $1\frac{3}{4}\%$ of 400?

3. What per cent of a peck is a quart?

4. A baseball player had 2 hits out of 5 times at bat in one game. What was his batting average for the game?

5. Dave paid \$36 for a bicycle which he later sold for \$30. Find his per cent of loss based on the selling price.

6. Electric toasters listed at \$9.60 are sold at a discount of $16\frac{2}{3}\%$. What is the selling price?

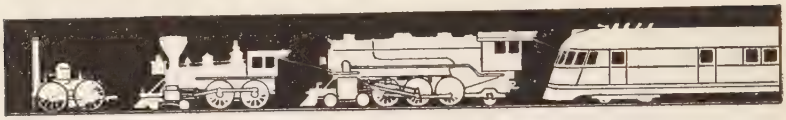
7. Mrs. Taylor bought for \$21 a chair that had been priced at \$24. What was the per cent of discount?

8. One month, Mr. Boyd sold 12 lots at \$600 each. What was the amount of his commission at 5%?

9. A basketball team lost 4 games, or 20% of the total number of games played. How many games had the team played?

10. A music store bought a banjo for \$12 and sold the instrument at a price which allowed 20% for overhead and 10% for profit, based on the cost. What was the profit? The selling price?

11. At what price must Mr. Crane sell electric washers that cost \$40 in order to allow 30% for overhead and a profit of 20%, based on the selling price?



1. 7.125 + .96 + 8.9794 + 3.998 + 6.4 + 4.928 + .8439

2. $6.09 - 4.6192$

3. $16.35 \times .0798$

4. $5.43\overline{)37.27}$

5. Divide $6\frac{2}{3}$ by 15.

6. $3\frac{1}{4} + 8\frac{2}{5} + 4\frac{1}{2} = ?$

7. $6\frac{15}{16} \times 8\frac{2}{3} = ?$

8. $5\frac{1}{2}\%$ of 420 = ?

9. 15 = $\frac{3}{4}\%$ of ?

10. \$42 = ? % of \$1600

11. At a sale, a radio sold for \$85. If the discount at the sale was 32%, what had been the regular price?

12. A dealer bought blankets at \$8.47 each. At what price must he sell them in order to include allowances of 18% for overhead and 12% for profit, based on the selling price?

13. Bob bought a used rowboat for \$32. After spending \$6.40 for repairs and \$7.60 for new oars, he sold the boat for \$52.90. What was his per cent of profit, based on the cost?

14. A commission merchant sold a load of peaches for \$459.63. His commission was 5%; freight charges were \$29.72; storage, \$18.80; and hauling, \$13.60. How much were the net proceeds to the peach grower?

15. Write an invoice for the following shipment of goods: 12 baseballs @ \$1.50; 3 sets of boxing gloves @ \$12.50 per set; 36 softball bats @ \$1.35. Discounts, 30% and 5%. Terms: 2%, 10 da.; net, 30 da. The bill is paid at the end of 30 da. Supply names, dates, and all other necessary information.

*Record your score on your Progress Chart. See page 321.

Do you know how the bank helps the business man, the manufacturer, the farmer, and the individual?

Banks serve the community in many ways. **Commercial banks,** so called because they help in the transaction of business, provide a convenient means for the transfer of money through **deposits** and **checks.**

A *deposit* is a sum of money in the form of bills, coins, or checks, placed in the care of a bank. A *check* is a written order on a bank for the payment of money on deposit.

Banks pay out money on deposit as ordered, and collect money for checks which are deposited with them.

Savings banks are banks in which people deposit money they wish to save. Most commercial banks have **savings departments.** These banks pay depositors of savings accounts a small per cent for the use of the money on deposit.

To help pay expenses and to make a profit, banks lend part of the money on deposit, charging for its use a small per cent of the amount loaned.

Mr. Henderson fills out a **deposit slip** each time he deposits money or checks in the bank. He gives the totals of the bills and coins, and lists each check separately. If a check is drawn on a bank in his own town, he enters the name of the bank. If it is drawn on a bank in another city, he writes the name of the city or the bank's number which is printed on the check.

DEPOSITED IN
THE MARINE TRUST COMPANY
OF BUFFALO

TO THE CREDIT OF R. B. Henderson

DATE DEPOSITED April 17 19--

	DOLLARS	CENTS
BILLS	76	00
SILVER	3	85
CHECKS AS FOLLOWS		
Liberty Bank	24	13
Rockland, Me.	112	50
60-620	5	72
TOTAL	222	20

A DEPOSIT SLIP

Mr. Henderson pays most of his bills by check, making a record of each payment on the **stub** of the check in his checkbook.

NO. 182
May 5 19--
TO Fisher Boat Co.
FOR Bill of 5/1

BAL. BRO'T FOR'D	560	04
AM'T DEPOSITED	61	90
" "	222	20
TOTAL	844	14
AM'T THIS CHECK	57	85
BAL. CAR'D FOR'D	786	29

STUB

THE MARINE TRUST COMPANY OF BUFFALO
SOUTH SIDE OFFICE 2213 SENECA ST.
10-2
S-S

BUFFALO, N. Y. May 5 19--

PAY TO THE ORDER OF Fisher Boat Co. $57 85/100

Fifty-seven and 85/100 DOLLARS

NO. 182

R. B. Henderson

CHECK

1. Tell what is shown on the check and the stub.
2. Why is it better to pay bills by check than by cash?
3. What is a **bankbook,** or **passbook?** Why is it used?
4. Secure several deposit slips and blank checks, and fill them in. Do not sign the checks. Why?

In order to cash a check made out to him, to deposit it, or to use it in paying someone else, Mr. Henderson **endorses** the check across the back at the left end.

THREE WAYS OF ENDORSING A CHECK

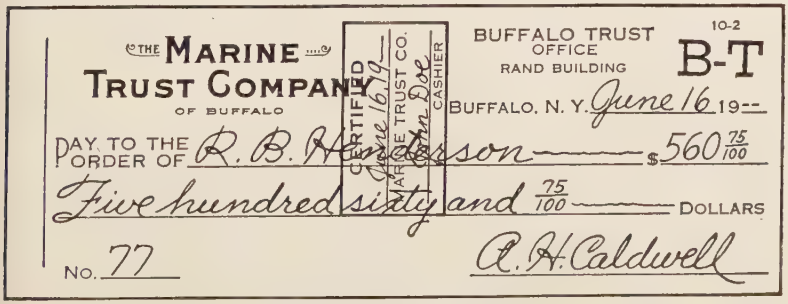
R. B. Henderson	For deposit R. B. Henderson	Pay to the order of D. C. Morgan R. B. Henderson

1. Which form of endorsement does Mr. Henderson use if he wishes to cash a check? If he wishes to deposit a check? If he wishes to transfer a check to someone else?

CERTIFIED CHECKS

One day, a customer offered Mr. Henderson his personal check. Mr. Henderson did not know the man, and requested that he pay with a **certified check.**

A certified check is a check the payment of which is guaranteed by the bank upon which it is drawn. Across the face of the check, an officer of the bank stamps the word **Certified** and the name of the bank, and writes in the date and his signature. When a check has been certified, the amount is at once deducted from the depositor's account. In order to have a check certified, a depositor must have sufficient funds in his account to pay the check.



A CERTIFIED CHECK

From time to time, Mr. Henderson receives from the bank a statement of his account which shows his last balance, his deposits and the sums drawn out since his last statement was made up, any charges for bank services, and his new balance. This new balance may not agree with the balance shown in Mr. Henderson's checkbook, for some of the checks he drew may not yet have been received by the bank for payment. With the statement, Mr. Henderson also receives his checks which have been returned to the bank and which the bank has stamped "Paid" or "Canceled."

1. Study the following portion of Mr. Henderson's bank statement. On the left are shown his checks which have been paid and the dates the bank paid them. On the right are shown Mr. Henderson's deposits and the dates on which they were made.

Mr. R. B. Henderson
1750 Grand Ave.
Buffalo, N.Y.

IN ACCOUNT WITH

THE MARINE TRUST COMPANY

Please examine at once. If no difference is reported in ten days, the account will be considered correct.

DATE	CHECKS IN DETAIL			DATE	DEPOSITS
BALANCE BROUGHT FORWARD ☛				Feb. 1, 19—	568.40
FEB. 2, 19—	50.00	28.60			
FEB. 5, 19—	87.40				
FEB. 8, 19—	17.50	214.88	1.65	FEB. 8, 19—	217.25
FEB. 9, 19—	137.88				
FEB. 11, 19—	47.28				
FEB. 14, 19—	16.35	121.62		FEB. 15, 19—	65.92

2. On Feb. 28, Mr. Henderson's bank balance was $627.12. His checkbook showed a balance of $491.36. What was the amount of the checks still unpaid by the bank?

1. In cashing a check at a bank how should a person endorse it? Such an endorsement, called a **blank endorsement,** could be used to transfer a check to someone else. Is it wise to use a blank endorsement for this purpose? Explain.

2. Tell some situations in which it would be wise to ask for certified checks.

3. Why is it important for a depositor to keep a record of his deposits and withdrawals in his checkbook?

4. Why should a depositor check his bank statement as soon as he receives it? How should he check it?

5. Find out what happens when a depositor overdraws his account; that is, draws a check for more funds than he has in the bank.

6. You make a deposit of 5 ten-dollar bills, 9 quarters, and a check for $88.85. Make and fill in a deposit slip.

7. James Pine had a bank balance of $968.20 on Feb. 10. On that day, he drew check No. 61 for $31.88, payable to Charles Thorn; and check No. 62 for $363.15, payable to Grant Owen. Make out the two checks and stubs.

8. Mr. Lamont had $387.49 in the bank on the first day of the month. During the month, he made deposits of $75, $92.85, $26.59, and $149.76. He drew checks for the following amounts: $286, $7.53, $29.80, $57.60. How much had he in the bank at the end of the month?

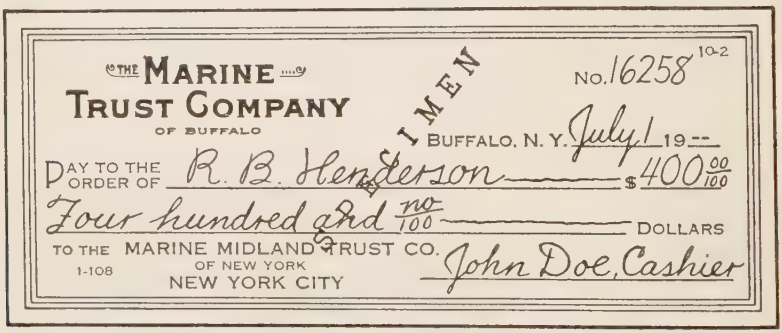
9. On Monday, February 1, a storekeeper had a balance of $375.80 in the bank. During the week, he deposited the following amounts: $22.30, $48.00, $12.60, $18.55, $43.74. During the same week, he drew checks for $25.80, $19.56, $76.75, $123.00, and $49.30. What was his bank balance at the close of the week?

One day, Mr. Henderson needed to order some goods from a New York firm. He was not well known to the firm, so instead of sending his personal check he went to his bank and bought a **bank draft** on a New York bank.

A *bank draft* is a check drawn by one bank against funds which it has on deposit in another bank.

As a matter of convenience, most banks have money on deposit in banks in other cities and have arrangements whereby these so-called **correspondent banks** handle business matters for them when necessary.

Bank drafts are accepted without question. Usually, a small fee is charged for a draft.


THE MARINE TRUST COMPANY
OF BUFFALO
I MEN
No. 16258 10-2
BUFFALO, N. Y. July 1 19--
PAY TO THE ORDER OF R. B. Henderson $400 00/100
Four hundred and no/100 DOLLARS
TO THE MARINE MIDLAND TRUST CO. OF NEW YORK NEW YORK CITY
1-108
John Doe, Cashier

A BANK DRAFT

Mr. Henderson had the bank draft made payable to him so he could endorse it to Black & Company and thus be able to prove that he had made payment. What endorsement would he use?

1. Compare the above bank draft with the check on page 360. How does it differ from a personal check?

2. Find the cost of drafts for the following amounts plus a fee of $\frac{1}{10}\%$: \$640; \$1200; \$895; \$2650.

"When you mail a check to a person in another town, how does it get back to your bank for payment?" Fred Henderson asked his father.

"There is nothing difficult about that," his father replied. "Here is how it works.

"I draw a check on the Marine Trust Co. of Buffalo and send it to J. W. Parker in Philadelphia. Mr. Parker endorses the check so it will be payable to the Dietz Supply Co. of Philadelphia in payment of a bill he owes them. The Dietz Supply Co. deposits the check in their Philadelphia bank. The Philadelphia bank credits the amount of the check to the Dietz Co. and sends the check for collection to the Buffalo bank that serves as its correspondent bank. The correspondent bank sends the check to what is known as the Buffalo **clearinghouse.** There it is turned over to the Marine Trust Co., which pays the check, cancels it, and returns it to me."

A clearinghouse is an association of banks in a community for the purpose of making a daily exchange of checks and drafts which are drawn on one another. In most places, one bank acts as the clearinghouse for them all. Each day, the banks send to the clearinghouse the checks they have received against one another. The total of the checks against each bank is figured, and the differences are charged or credited to the several banks.

1. Bring to class a paid check, and see if you can describe its travels from the bank endorsements on the back.

2. Make a diagram to illustrate the travels of the check Mr. Henderson sent to Mr. Parker.

3. Would Mr. Henderson's check have had to go through the clearinghouse if the Marine Trust Co. of Buffalo had been the correspondent bank of the Philadelphia bank? Explain.

See if you can do each of these tests in the time given.

SET 1. ADDITION (4 minutes after copying)

1. $369 + 462 + 358 + 809 + 847 + 769 + 863 + 957$
2. $565 + 196 + 726 + 887 + 733 + 254 + 992 + 889$
3. $775 + 565 + 354 + 654 + 788 + 937 + 929 + 421$
4. $8.6 + 39.9 + 9.89 + 48.4 + 1.164 + 7.07 + 8.319$
5. $77.84 + .85 + 5.056 + .89 + 3.896 + 54.16 + .33$

SET 2. SUBTRACTION (8 minutes after copying)

	A	B	C
1.	$50786 - 20940$	$\$1729.43 - \$1273.51$	$6843.3 - 3082.7$
2.	$85564 - 36199$	$\$7321.74 - \$6585.46$	$1608.03 - 876.34$
3.	$75285 - 17427$	$\$4723.17 - \$896.35$	$87.435 - 17.59$
4.	$94414 - 28674$	$\$6372.02 - \$4994.98$	$11.49 - 2.0612$

SET 3. MULTIPLICATION (20 minutes after copying)

1.	526×2493	$143 \times \$371.60$	$.045 \times 915.2$
2.	893×9025	$560 \times \$815.67$	$.048 \times 403.8$
3.	689×4627	$789 \times \$154.38$	20.1×971.58
4.	786×8976	$407 \times \$430.89$	7.95×8.076

SET 4. DIVISION (35 minutes after copying)

1.	$16975 \div 25$	$\$546.05 \div 815$	$6.513 \div .78$
2.	$21821 \div 38$	$\$157.76 \div 232$	$.4281 \div .065$
3.	$20704 \div 72$	$\$506.39 \div 641$	$27.302 \div 2.9$
4.	$33205 \div 57$	$\$466.56 \div 486$	$727.42 \div 84.3$
5.	$46354 \div 86$	$\$800.47 \div 383$	$36.197 \div 3.75$

Business concerns and others often borrow money and pay for the use of it. Mr. Henderson may borrow money to enlarge his business. Mr. and Mrs. Larson may borrow to build a home. Howell & Company may borrow in order to buy new machinery or to carry on their business during a dull season until trade picks up again.

A moneylender of the Middle Ages.

Banks are usually willing to lend money to responsible people, charging a per cent of the amount that is loaned. The borrower, of course, must be able to show that he can repay the money when it is due.

Money that is paid for the use of money is called *interest.*

The per cent of interest paid is called the *rate of interest.* Many states fix by law a *legal rate* of interest, which is the highest rate that can be charged. The lender, however, may charge a lower rate than the legal rate if he desires.

The period for which interest is paid is called the *time.*

The amount of money on which interest is paid is called the *principal.*

The sum of the principal and the interest is called the *amount.*

Interest is based on a yearly rate. Interest at 5% means 5% a year. Interest payments may be required monthly, quarterly, semiannually, or annually.

How to Find Interest

1. Find the interest on \$1200 at 4% for 1 yr.

\$1200 principal
.04 rate
\$48.00 interest for 1 yr.

Multiply the principal by the rate of interest expressed as a decimal.

To find the interest for a number of years, multiply the interest for one year by the number of years.

---

2. Find the interest on \$960 at 6% for 2 mo. 24 da.

$6\% = \frac{6}{100}$

Change the rate of interest to a fraction.

2 mo. 24 da. $= \frac{84}{360}$ yr.

Change the months and days to a fraction of a year. Count each month as 30 da. Count 360 da. as a year.

$\$960 \times \frac{\cancel{6}^{1}}{\cancel{100}_{25}} \times \frac{\cancel{84}^{21}}{\cancel{360}_{\cancel{6}}} = \$13.44$ int. for 2 mo. 24 da.

Multiply the principal by the rate by the time. Cancel when possible.

---

3. Find the interest on \$750 at 5% for 2 yr. 3 mo. 18 da.

\$750 × .05 = \$37.50 int. for 1 yr.
\$37.50 × 2 = \$75 int. for 2 yr.
3 mo. 18 da. $= \frac{108}{360}$ yr.

$\$\cancel{37.50}^{3.75} \times \frac{\cancel{108}^{3}}{\cancel{360}_{\cancel{10}}} = \$11.25$ int. for 3 mo. 18 da.

\$75 + \$11.25 = \$86.25 int. for 2 yr. 3 mo. 18 da.

Find the interest for the number of years.

Change the months and days to a fraction of a year, and find the interest for that time.

Find the total interest.

To find the amount, add the principal and the interest.

Find the interest and the amount of each of the following. (In figuring interest, when the answer contains a fraction of $\frac{1}{2}¢$ or more, count it as $1¢$. Drop any fraction less than $\frac{1}{2}¢$.)

	Prin.	Rate	Time		Prin.	Rate	Time
1.	$325	5%	1 yr.	9.	$840	6%	4 mo. 15 da.
2.	$250	6%	2 yr.	10.	$620	4%	1 mo. 12 da.
3.	$900	3%	4 yr.	11.	$1800	5%	8 mo. 6 da.
4.	$788	$4\frac{1}{2}$%	1 yr.	12.	$960	6%	4 mo. 12 da.
5.	$1400	4%	3 yr.	13.	$240	$3\frac{1}{2}$%	1 yr. 6 mo.
6.	$345	4%	3 mo.	14.	$160	6%	1 yr. 3 mo. 15 da.
7.	$72	6%	10 mo.	15.	$524	4%	2 yr. 1 mo. 6 da.
8.	$186	3%	2 mo.	16.	$1600	$4\frac{1}{2}$%	2 yr. 4 mo. 18 da.

17. The Medford Manufacturing Company borrowed $4800 at $4\frac{1}{2}$% for 6 mo. 18 da. How much interest should the company pay?

FINDING THE TIME BETWEEN DATES

Find the time from November 17, 1939, to October 11, 1941.

This example is worked in the box at the right. Explain each step.

Yr.	Mo.	Da.
1941	10	11
1939	11	17
1	10	24

Find the time between the dates in each of the following:

1. From May 16, 1940, to August 28, 1941.

2. From September 30, 1940, to December 4, 1941.

3. From January 26, 1938, to September 15, 1942.

4. From July 15, 1939, to May 12, 1941.

5. Mr. Kennedy borrowed $2400 at 6% on April 14, 1940. He repaid the loan on October 15, 1941. What amount was due at the time of payment?

Interest may always be found by the following short rule when the time is expressed as years or a fraction of a year:

Interest = principal × rate × time.

Using i for interest, p for principal, r for rate, and t for time, the rule may be written:

$$i = p \times r \times t \quad \text{or} \quad i = prt$$

When letters and signs or letters, signs, and figures are used to express a rule, we call the expression a *formula*. When letters are written together in a formula, as *prt*, their number values are to be multiplied.

How to Find Interest by Using the Formula

Mr. Snyder borrows \$1125 at 4% interest for 6 mo. 18 da. Find the interest.

$i = prt$ — Write the formula.

$p = \$1125 \quad r = \frac{4}{100} \quad t = \frac{198}{360}$ — Substitute the known number values for the letters in the formula.

$i = \cancel{\$1125}^{\,9} \times \frac{\cancel{4}}{\cancel{100}_{\,\cancel{4}}} \times \frac{\overset{11}{\cancel{198}}}{\cancel{360}_{\,\cancel{20}\;4}} = \$24.75$ interest — Multiply the principal by the rate by the time.

Using the interest formula, find the interest on:

	Prin.	Rate	Time		Prin.	Rate	Time
1.	\$200	6%	3 mo.	5.	\$520	2%	2 yr. 6 mo.
2.	\$160	5%	9 mo.	6.	\$80	$4\frac{1}{2}$%	96 da.
3.	\$196	3%	60 da.	7.	\$366	3%	4 mo. 6 da.
4.	\$2100	$4\frac{1}{2}$%	72 da.	8.	\$546	6%	5 mo. 21 da.

9. Mr. Reed lent \$2400 at 5% for 4 mo. and \$2400 at 4% interest for 5 mo. Which loan earned the greater interest? How much greater?

Ordinarily, interest is figured on a 360-da. year. The Treasury Department of the United States, some state governments, and some banks, however, figure interest on 365 da. for an ordinary year and 366 da. for a leap year.

Interest figured on the *exact number of days in a year* is called *exact interest.*

How to Find Exact Interest

Find the exact interest on a bank loan of \$4800 for 45 da. at 2% interest.

$2\% = \frac{2}{100}$ Change the rate of interest to a fraction.

$45 \text{ da.} = \frac{45}{365}$ yr. Change the number of days to a fractional part of a year, using the exact number of days in the year as the denominator.

$\$4800 \times \frac{2}{100} \times \frac{9}{\frac{45}{365}{}_{73}} = \$11.84$ exact interest

Multiply the principal by the rate by the time.

Find the exact interest on:

1. \$730 at 5% for 128 da.; for 192 da.
2. \$1200 at 6% for 146 da.; at 4%.
3. \$2400 at 6% for 96 da. in a leap year.
4. \$1845 at 6% for 85 da.; for 219 da.
5. Find the difference between exact and ordinary interest on \$1460 at 6% for 90 da.

Remember: In working interest problems, always figure the interest on 360 da. in a year unless otherwise instructed.

1. Some business men lent Mr. Owens \$3000 at 4% interest, payable semiannually, to buy new bottling equipment for his dairy. At the end of $3\frac{1}{2}$ yr., Mr. Owens repaid the loan. How much interest did he pay during the $3\frac{1}{2}$ yr.?

2. Mr. Brannock borrowed \$1800 at 5% interest to open a small bookshop. He repaid the loan with interest at the end of 1 yr. 4 mo. 15 da. How much was the interest? What amount did he pay?

3. The owner of the Star Wall Paper & Paint Co. decided to enlarge his shop. He borrowed \$2400 at $5\frac{1}{2}$% on Feb. 15 and repaid the amount with interest on Jan. 30 of the following year. Find the interest and the amount.

4. Mr. Bliss decided to sell automobile supplies at his public garage. To pay for the merchandise, he borrowed \$460 at 6%. He repaid the loan with interest at the end of 10 mo. 12 da. How much did he pay?

5. When Mr. Burke opened his new florist shop, he used most of his funds to equip his store. To pay for a delivery truck, he borrowed \$675 on July 7 at 6% interest. He repaid the loan on Apr. 5 of the following year. Find the amount he had to pay.

SET 1. ADDITION

1.
18573
36963
52954
82
6290
21
7559

2.
652.57
4.43
180.984
1.98
5.098
.93
377.34

3. $\frac{3}{4} + \frac{5}{8} + \frac{7}{24} = ?$

4. $3\frac{1}{5} + 9\frac{4}{15} + 5\frac{7}{30} = ?$

5. $8\frac{1}{4} + 3\frac{2}{5} + 7\frac{5}{8} = ?$

SET 2. SUBTRACTION

1. 33014 − 18423

2. 91891 − 43896

3. 802. − 760.191

4. 908.11 − 54.489

5. $\frac{5}{12} - \frac{2}{15} = ?$

6. $86 - 5\frac{5}{6} = ?$

7. $21\frac{2}{3} - 16\frac{7}{8} = ?$

SET 3. MULTIPLICATION

1. 6258 × 106

2. 47130 × 120

3. 652.98 × 3.45

4. 98047 × .6789

5. $54 \times \frac{5}{9} = ?$

6. $\frac{7}{24} \times \frac{3}{8} = ?$

7. $12\frac{3}{4} \times 6\frac{5}{6} = ?$

8. Multiply 864 by 50; 25; 5; $12\frac{1}{2}$; $33\frac{1}{3}$; $16\frac{2}{3}$; 125.

SET 4. DIVISION

1. $43\overline{)37668}$

2. $548\overline{)35072}$

3. $152\overline{)\$164.54}$

4. $.26\overline{)652.6}$

5. $6.7\overline{)27.526}$

6. $.945\overline{)137.97}$

7. $\frac{7}{12} \div \frac{9}{16} = ?$

8. $126 \div \frac{7}{8} = ?$

9. $3\frac{3}{5} \div 6\frac{3}{8} = ?$

10. Divide 1235 by 25; 50; $16\frac{2}{3}$; $12\frac{1}{2}$; 125; $33\frac{1}{3}$.

THE 60-DAY OR 6% METHOD OF FINDING INTEREST

The interest on any principal at 6% for 1 yr. of 360 da. is 6% of the principal. The interest on \$400 at 6% for 360 da. is \$24. Sixty days is $\frac{1}{6}$ of 360 da. Therefore, the interest on \$400 at 6% for 60 da. is $\frac{1}{6}$ of \$24, or \$4, which is 1% or .01 of \$400, the principal.

To find the interest on any principal at 6% for 60 da., move the decimal point in the principal two places to the left.

Six days is what part of 60 da.? If the interest for 60 da. is \$4, what is the interest for 6 da.? How would you find the interest for 30 da.? 20 da.? 15 da.? 90 da.?

How to Find the Interest at 6% by the 60-Day Method

Find the interest on \$1725 for 81 da. at 6%.

81 da. = 60 da. + 15 da. + 6 da.

Separate the time into periods of 60 da. and simple parts thereof.

The interest on \$1725 at 6% for:

60 da. = \$17.25

15 da. = 4.3125 ($\frac{1}{4}$ **of \$17.25**)

6 da. = 1.725 ($\frac{1}{10}$ **of \$17.25**)

81 da. = \$23.2875 = \$23.29

Find the interest for 60 da. by moving the decimal point in the principal two places to the left.

Find the interest for the other periods by multiplying the interest for 60 da. by the fractional part of 60 da. which each period equals.

Add the amounts of interest.

Using the 60-day method, find the interest at 6% on:

1. \$665 for 60 da.
2. \$962 for 90 da.
3. \$740 for 30 da.
4. \$336 for 75 da.
5. \$210 for 16 da.
6. \$780 for 9 da.
7. \$426 for 80 da.
8. \$528 for 90 da.
9. \$860 for 48 da.
10. \$240 for 3 da.
11. \$225 for 66 da.
12. \$542 for 72 da.

3% is $\frac{1}{2}$ of 6%. Since the interest at 6% on \$185.60 for 60 da. is \$1.856, the interest at 3% is $\frac{1}{2}$ of \$1.856, or \$.93.

What would be the interest at $3\frac{1}{2}$% on \$185.60 for 60 da.? $3\frac{1}{2}\% = \frac{3\frac{1}{2}}{6}$ or $\frac{7}{12}$ of 6%. $\frac{7}{12}$ of \$1.856 = \$1.0826. The interest on \$185.60 at $3\frac{1}{2}$% for 60 da. is \$1.08.

Remember: To find the interest by the 60-day method at a rate other than 6%, first find the interest at 6%. Then find what fractional part of 6% the given rate is, and multiply the interest at 6% by this fraction.

In finding interest by the 60-day method, find each interest item to four decimal places if necessary, and keep all decimal places until the total interest is found. Then express this interest to the nearest cent.

Find the interest on the following by the 60-day method:

	Prin.	Rate	Time		Prin.	Rate	Time
1.	\$800	3%	66 da.	5.	\$746	4%	78 da.
2.	\$540	4%	54 da.	6.	\$184.20	3%	75 da.
3.	\$280	4%	45 da.	7.	\$3600	2%	96 da.
4.	\$630	5%	72 da.	8.	\$4200	$4\frac{1}{2}$%	24 da.

When finding interest by the 60-day method, it is sometimes possible to shorten the work by using subtraction. Study the following short way of finding the interest on \$720 at 6% for 114 da.:

114 da. = 120 da. − 6 da.

Interest on \$720 at 6% for:

120 da. = \$14.40 (2 × \$7.20)

6 da. = .72 ($\frac{1}{10}$ of \$7.20)

114 da. = \$13.68

Using the short method, find the interest on:

9. \$1280 at 6% for 54 da.
10. \$82.45 at 6% for 48 da.
11. \$2340 at 4% for 108 da.
12. \$126.50 at 3% for 174 da.

Because of the large number of loans on which banks must figure interest each day, bankers find it easier to work from printed interest tables similar to the one below. There are separate interest tables for the different interest rates. The tables which banks use show the interest for all periods up to and including 1 yr. In the table below, the interest figures have been rounded to three decimal places.

TABLE OF SIMPLE INTEREST AT 6%
(Based on 360 days in a year)

Days	$100	$200	$300	$400	$500	$600	$700	$800	$900
1	.017	.033	.05	.067	.083	.10	.117	.133	.15
2	.033	.067	.10	.133	.167	.20	.233	.267	.30
3	.050	.100	.15	.200	.250	.30	.350	.400	.45
4	.067	.133	.20	.267	.333	.40	.467	.533	.60
5	.083	.167	.25	.333	.417	.50	.583	.667	.75
6	.100	.200	.30	.400	.500	.60	.700	.800	.90
7	.117	.233	.35	.467	.583	.70	.817	.933	1.05
8	.133	.267	.40	.533	.667	.80	.933	1.067	1.20
9	.150	.300	.45	.600	.750	.90	1.050	1.200	1.35
10	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50
20	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
30	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
60	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
90	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50
120	2.000	4.000	6.00	8.000	10.000	12.00	14.000	16.000	18 00

To find the interest on \$500 at 6% for 30 da., first find 30 da. in the **Days** column. Following across to the column headed **\$500,** you find that the interest is \$2.50.

50 da. = 30 da. + 20 da. How would you find the interest on \$700 at 6% for 50 da., using the above table? \$3.50 + \$2.333 = \$5.833, or \$5.83.

Using the table, find the interest on \$80 at 6% for 90 da. \$80 is $\frac{1}{10}$ of \$800. Find the interest on \$800, and move the decimal point one place to the left. Why? What is the interest?

\$3000 is 10 times \$300. How would you find the interest on \$3000 at 6% for 7 da., using the table? $10 \times \$.35 = ?$

Study the following example, and explain how each separate amount of interest was obtained from figures given in the interest table opposite.

Find the interest on \$450 at 6% for 37 da.

\$450 = \$400 + \$50 37 da. = 30 da. + 7 da.

The interest on: \$400 for 30 da. = \$2.00
\$50 for 30 da. = .25
\$400 for 7 da. = .467
\$50 for 7 da. = .0583

Interest on \$450 at 6% for 37 da. = \$2.7753 = \$2.78

Remember: In using an interest table, always retain all decimal places until the final answer.

Using the interest table, find the interest at 6% on each of the following:

	Principal	Time		Principal	Time
1.	\$800	30 da.	6.	\$80	14 da.
2.	\$300	20 da.	7.	\$560	38 da.
3.	\$5000	5 da.	8.	\$1050	27 da.
4.	\$2000	13 da.	9.	\$480	58 da.
5.	\$70	35 da.	10.	\$240	49 da.

Use the interest table in working these problems:

11. Find the interest on a loan of \$1300 at 6% for 127 da.

12. Mr. Carroll borrowed \$2700 at 6% for 79 da. How much did he owe at the end of that time?

13. Mr. Keene borrowed \$5900 at 6% interest for 5 mo. 20 da. Find the amount Mr. Keene paid when the loan became due.

What part of a year is 3 mo.? 4 mo.? If the interest on \$800 for 3 mo. is \$12, what is the interest for 1 yr.? Why? If the interest on \$800 for 4 mo. is \$16, what is the yearly interest? Why?

If we know what the interest is for 1 yr. on any principal, we can easily find the yearly rate of interest.

How to Find the Yearly Rate of Interest

Mr. Endicott lent \$1800 for 2 mo. and was paid \$12 interest. What yearly rate of interest was he paid?

Int. for 2 mo. or $\frac{1}{6}$ yr. = \$12

Int. for 1 yr. = 6 × \$12, or \$72

\$72 is what % of \$1800?

\$72 ÷ \$1800 = .04 = 4% rate of int.

First, find how much the interest would be for 1 yr.

To find the yearly rate of interest, divide the interest for 1 yr. by the principal, and express the result as a per cent.

Remember: To find the yearly rate of interest, first find how much the interest would be for 1 yr. Then divide this interest by the principal, and express the result as a per cent.

Find the yearly rate of interest on each of the following:

	Prin.	Time	Interest		Prin.	Time	Amount
1.	\$600	1 yr.	\$45	6.	\$1000	6 mo.	\$1027.50
2.	\$400	6 mo.	\$10	7.	\$1200	3 yr.	\$1380
3.	\$800	2 mo.	\$6	8.	\$1500	1 mo.	\$1505
4.	\$250	3 mo.	\$3.75	9.	\$4000	60 da.	\$4030
5.	\$5400	4 mo.	\$63	10.	\$620	90 da.	\$629.30

11. A loan of \$360 was settled after 9 mo. by the payment of \$376.20. What yearly rate of interest was paid?

1. If the interest on a loan of \$3550 for 1 yr. is \$142, what is the rate of interest?

2. Ralph borrowed \$600 to help pay for his course in an agricultural college. Two years later, he repaid the loan with $4\frac{1}{2}\%$ interest. How much did he pay?

3. Mr. Green borrowed \$655 at 6% interest to build a silo. He repaid the loan with interest at the end of 3 mo. How much did he pay?

4. A market gardener borrowed \$450 at 6% on Mar. 15 to pay for seed and fertilizer. He repaid the loan with interest on Oct. 21. How much did he pay?

5. Mr. Long lent a friend \$600. The friend agreed to pay \$618 in settlement of the loan in 8 mo. What rate of interest did he agree to pay?

6. A man owes \$100 and is given the choice of paying it in 84 da. with interest at 6%, or of paying \$8.50 a week for 12 wk. Which is the cheaper plan? How much will he save by it?

7. Mr. Shelton lends \$1800 for 8 mo. at $4\frac{1}{2}\%$ and \$1600 for 6 mo. at 6%. Which loan earns more interest? How much more?

8. James Wells bought some goods for \$916, paying $\frac{1}{4}$ cash, and agreeing to pay the balance in 4 mo. with interest at 5%. How much was his second payment?

9. Mrs. Locke owed a bill of \$122.16 which was 5 mo. overdue. When she paid the bill, she was charged $\frac{1}{2}\%$ per month for the overdue time. How much did she have to pay to settle the account?

Work the examples in column A. If you fail on any example, do the example opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review Pages
1. Finding Interest for Years, Months, and Days		
Prin. Rate Time	Prin. Rate Time	
\$560 4% 2 yr.	\$840 5% 3 yr.	368–
\$1800 6% 6 mo. 12 da.	\$1200 6% 3 mo. 15 da.	369
\$2000 4% 2 yr. 9 mo. 6 da.	\$3600 5% 1 yr. 6 mo. 12 da.	
2. Finding Interest, Using the Interest Formula		
p *r* *t*	*p* *r* *t*	
\$64 6% 5 mo.	\$120 4% 10 mo.	370
\$128 3% 7 mo. 12 da.	\$540 5% 8 mo. 18 da.	
3. Finding Exact Interest		
Prin. Rate Time	Prin. Rate Time	
\$472 6% 215 da.	\$365 4% 84 da.	371
\$1800 5% 45 da.	\$1450 3% 146 da.	
4. Finding Interest by the 60-Day Method		
Prin. Rate Time	Prin. Rate Time	
\$625 6% 90 da.	\$454 6% 30 da.	374
\$980 3% 75 da.	\$780 4% 45 da.	375
\$2560 $4\frac{1}{2}$% 183 da.	\$1200 4% 129 da.	
5. Finding the Yearly Rate of Interest		
Prin. Time Int.	Prin. Time Int.	
\$1500 90 da. \$7.50	\$900 6 mo. \$36	378
\$240 72 da. \$4	\$390 5 mo. \$6.50	

John Dexter borrowed $800 from Louis Nichols to meet an emergency payment, giving his written promise to repay the loan with interest at the end of six months.

A written promise for the payment of money is called a *promissory note* or simply a *note*.

$ 800.00

Richmond, Va. April 10, 19--

Six months ---------- after date I promise to pay to the order of ----------Louis Nichols---------- Eight hundred and no/100---------- Dollars with interest at 6%.

Value received John Dexter

A PROMISSORY NOTE

Notes may differ in wording, but all notes should contain: (1) the **date** and the **place** where the note is made; (2) the **term,** or time the note is to run; (3) the words **promise to pay;** (4) the name of the **payee,** or person to whose order the money is to be paid; (5) the **face,** or amount for which the note is written; (6) the signature of the **maker,** or person who signs the note; and (7) the words **value received** to show that the maker has received the money or its value in other things.

1. Name the place, the date, the term, the payee, and the maker in the above note.

A note which states the time the note is to run is a **time note.** Time notes are usually given for a number of days, months, or years, and are payable at the end of the stated time, although payment may be made sooner if the maker desires. The above is a time note. Time notes sometimes give the **date of maturity,** or date when the note is due.

Some notes bear the words "On demand" instead of the time the note is to run. **Demand notes** are payable with interest whenever the lender requests payment, or sooner if the maker desires. On demand notes, interest is usually payable quarterly.

A note which bears the words **with interest** is called an **interest-bearing note.** If no specific rate is stated, the legal rate of interest must be paid.

A note which does not include the words **with interest** is called a **non-interest-bearing note.** On such notes, only the face of the note is payable.

Notice that the note on page 381 is made out **to the order of** Louis Nichols. This means that the note is payable to Mr. Nichols or to any person to whom he may transfer or sell it. A note that can be bought or sold is called a **negotiable note.**

A note which is payable only to a specified person cannot be transferred or sold and is called a **non-negotiable note.**

When the payee sells or transfers a note, he endorses it on the back. The endorser of a note is responsible for its payment if the maker does not pay the note when it becomes due.

2. Give reasons why a person who lends money might prefer to have a negotiable note rather than one which is not negotiable.

Banks and others who lend money sometimes require the maker of a note to give a guarantee for its payment. The maker may: (1) have some responsible person guarantee payment by endorsing the note, or (2) put up bonds or other **securities** which may be sold for payment in case the note is not paid. Securities given in this way are called

collateral. If a note is paid when due, the securities are returned to the maker. Notes for which collateral is given differ from the ordinary promissory note. Collateral is usually requested on demand notes.

3. Get a blank collateral note from a bank, and tell the class how it differs from an ordinary note.

Write negotiable notes for the following, writing the word "Specimen" or "Sample" across each note. Why?

4. Albert Forster of Tulsa, Okla., agrees to pay $200 to Fred Merrill 30 da. after Jan. 7, with 5% interest.

5. Today, Roy Hess of Denver agrees to pay $500 to Sheedy & Co. 3 mo. after date, with interest at 6%.

6. Max Starr of San Francisco agrees to pay $375 to Clara B. Stearns on demand, with interest at $5\frac{1}{2}\%$.

The date of maturity of a note given for a number of months is found by counting forward the given number of months. A 4-mo. note dated Aug. 7 will be due Dec. 7.

To find the date of maturity of a note given for a number of days, count forward the exact number of days from the date of the note.

To find the date of maturity of a 60-day note dated May 6, think:

May, 25 da. left (31 da. − 6 da.)
June, 30 da.
July, 5 da. (60 da. − 55 da. in May and June)
60 da. The date of maturity is July 5.

7. Find the date of maturity of a 3-mo. note dated June 16; of a 30-da. note dated Mar. 16; of a 90-da. note dated July 17; of a 60-da. note dated Feb. 26.

In lending money on time notes, banks usually deduct the interest in advance and give the borrower the difference between the interest and the face of the note. This is called **discounting** the note. The amount deducted is called the **bank discount,** and the difference the borrower receives is called the **proceeds.**

Since interest on bank time-loan notes is deducted in advance, such notes do not bear the words **with interest** and are called non-interest-bearing notes.

How to Find the Bank Discount and the Proceeds of a Non-Interest-Bearing Note for the Full Term

Mr. Kenyon borrows $1250 from a bank, giving his 30-da. note. Find the bank discount at 6% and the proceeds.

Bank discount on $1250 at 6% for 60 da. = $12.50

Bank discount for 30 da. = $6.25

Find the bank discount on the face of the note at the given rate for the given term.

$1250.00 face of note
6.25 bank discount
$1243.75 proceeds

To find the proceeds, subtract the discount from the face of the note.

Find the bank discount and the proceeds of each of the following notes:

	Face	Int. Rate	Term		Face	Int. Rate	Term
1.	$450	6%	60 da.	5.	$1115	5%	3 mo.
2.	$780	6%	60 da.	6.	$386	7%	1 mo.
3.	$920	6%	90 da.	7.	$1540	6%	75 da.
4.	$1080	5%	30 da.	8.	$552	$4\frac{1}{2}$%	4 mo.

9. The Bankers Trust Company lent the Ajax Cleaner Co. $675 on a 45-da. note, discounting the note at 6%. Find the bank discount and the proceeds.

Promissory notes are sometimes given in business transactions as guarantees for the payment of goods. Such notes may or may not bear interest.

If the receiver of a note needs the money before the note matures, he can sell the note to a bank. In that case, the bank will deduct bank discount from the date the note is discounted to the date of maturity. This period of time is called the **term of discount.**

How to Find the Bank Discount and the Proceeds of a Non-Interest-Bearing Note for a Partial Term

Find the bank discount and the proceeds of a 60-da. note for $1800 dated May 6 and discounted May 12 at 6%.

60 da. from May 6 = July 5

May 12 to July 5 = 54 da.

Bank discount on $1800 at 6% for 54 da. = ?

Bank discount for:

60 da. = $18.00
— 6 da. = 1.80
54 da. = $16.20

$1800.00 face of note
16.20 bank discount
$1783.80 proceeds

Find the date of maturity.

Find the number of days between the date of discount and the date of maturity.

Find the bank discount on the face of the note for the term of discount.

To find the proceeds, subtract the bank discount from the face of the note.

Find the bank discount and the proceeds of these notes:

	Face	Date	Term	Rate of Disc.	Date of Disc.
1.	$400	Mar. 3	60 da.	6%	Mar. 9
2.	$390	May 18	3 mo.	6%	June 7
3.	$1860	Oct. 21	90 da.	5%	Nov. 14
4.	$640	Feb. 25	30 da.	5%	Mar. 3

Interest-bearing notes are frequently sold to a bank at a discount. The discount on such notes is reckoned on the amount due at maturity.

How to Find the Bank Discount and the Proceeds of an Interest-Bearing Note

On Aug. 11, an automobile dealer sold to a bank a 90-day note for $600, dated Aug. 5 and bearing 5% interest. The bank discounted the note at 6%. Find the bank discount and the proceeds.

$\$600 \times \frac{5}{100} \times \frac{90}{360} = \$7.50$ int.

Find the interest on the note.

$\$600 + \$7.50 = \$607.50$ amount due at maturity

Find the amount due at maturity.

90 da. from Aug. 5 = Nov. 3 date of maturity

Find the date of maturity.

Aug. 11 to Nov. 3 = 84 da. term of discount

Find the term of discount.

$607.50 due at maturity

$6.075 bank disc. for 60 da.
2.025 bank disc. for 20 da.
.405 bank disc. for 4 da.
$8.505 = $8.51 bank discount for 84 da.

Find the bank discount on the amount due at maturity at the given rate for the term of discount.

$\$607.50 - \$8.51 = \$598.99$ proceeds

To find the proceeds, subtract the bank discount from the amount due at maturity.

Remember: To find the bank discount on an interest-bearing note, first find the amount due at maturity (principal + interest); then find the bank discount on this amount at the bank's rate for the term of discount.

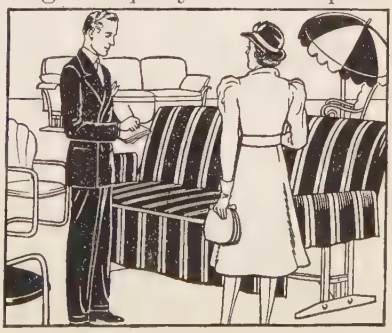
Find the date of maturity, the bank discount, and the proceeds of each of these interest-bearing notes:

	Face of Note	Term	Rate of Interest	Date of Note	Date of Discount	Rate of Discount
1.	\$1800	60 da.	6%	Sept. 10	Sept. 10	5%
2.	\$360	3 mo.	6%	Mar. 6	Mar. 6	6%
3.	\$420	2 mo.	5%	Apr. 9	Apr. 16	6%
4.	\$1200	90 da.	4%	Aug. 31	Sept. 6	6%
5.	\$2400	60 da.	5%	July 12	July 16	5%
6.	\$256	4 mo.	6%	Jan. 14	Jan. 15	6%
7.	\$368	30 da.	6%	May 1	May 10	6%
8.	\$240	60 da.	5%	Oct. 14	Oct. 17	6%

9. A customer gave Mr. Logan a 90-da. note for \$384, dated June 14, and bearing interest at 5%. On June 21, Mr. Logan's bank discounted the note at 6% and deposited the proceeds to his account. How much was the deposit?

10. Mr. North's balance at his bank was \$68.25. The bank discounted for him a 60-da. note for \$258, bearing 6% interest. The term of discount was 54 da., and the rate was 6%. Mr. North deposited the proceeds to his account. What was his bank balance then?

11. The Chase Manufacturing Company sold a department store 62 gliders @ \$12.50, less discounts of 40% and 10%, receiving a 30-da. note in payment, dated Feb. 17, and bearing 6% interest. The note was received Feb. 20 and was immediately discounted at 6%. Find the proceeds.



In finding interest periods, bankers use a table like the one below. It gives the number of days between any day in one month and the corresponding day in any following month.

From Any Day of	To the Same Day of Any of the Following Months											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

When the interest period includes Feb. 29 of a leap year, add one day.

To find the time between Mar. 10 and June 12, find March in the first column, and follow across to the column headed June. There are 92 da. from Mar. 10 to June 10. From June 10 to June 12 = 2 da. 92 da. + 2 da. = 94 da.

To find the time from Apr. 15 to Nov. 4, first find the time from Apr. 15 to Nov. 15, which is 214 da. Nov. 4 to Nov. 15 = 11 da. 214 da. − 11 da. = 203 da.

Using the table, find the number of days from:

1. Jan. 16 to Apr. 21

2. June 12 to Sept. 28

3. Aug. 14 to Mar. 3

4. May 15 to Jan. 12

Find the bank discount and the proceeds of the following:

5. $2400 from May 6 to July 13 at 6%.

6. $3250 from Dec. 15, 1940, to Mar. 12, 1941, at 5%.

On May 1, the Rankin Manufacturing Company of Philadelphia, Pa., sold Henry Parker of Newark, N. J., $1200 worth of goods. The sale was made "subject to a 60-day **trade acceptance.**"

A *trade acceptance* is a paper drawn by the seller on which the buyer acknowledges receipt of goods and agrees to pay for his purchase at a stated time.

When the Rankin Company shipped the goods, it mailed to Mr. Parker an invoice and a trade acceptance like that below in which the part in brackets was left blank.

TRADE ACCEPTANCE

No. 61 Philadelphia, Pa., May 1 19--

To Henry Parker Newark, N. J.

Sixty days after date Pay to the order of Rankin Mfg. Co.

Twelve hundred and no/100---------- Dollars (---$1200----)

Accepted at Newark N.J. on May 4 19--

Payable at First National Bank RANKIN MANUFACTURING CO.

Location Newark, N. J.

Henry Parker (Signature of Acceptor)

J. Rankin President (Signature of Drawer)

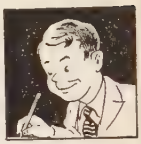
The transaction which gives rise to this instrument is the purchase of goods by the acceptor from the drawer.

When he received the goods, Mr. Parker accepted the trade acceptance by filling in the blanks at the lower left and signing his name. He then returned the acceptance to the Rankin Manufacturing Company. Like a promissory note, a completed trade acceptance can be held until due, or can be sold or discounted.

Find the proceeds of these acceptances discounted at 6%:

	Face	Date	Term	Date of Discount
1.	$650	Jan. 16	60 da.	Jan. 21
2.	$580	Feb. 14	90 da.	Feb. 17
3.	$1500	May 31	30 da.	June 9

Try to work these problems without using a pencil and paper. Write only the answers. If you cannot do a problem mentally, work it on paper.



1. Find the number of days between May 20 and June 26.

2. A note for 3 mo., dated Aug. 4, is due on what date?

3. What is the interest on \$600 for 60 da. at 6%?

4. Find the interest on \$500 for 1 yr. at 6%.

5. What is the interest on \$800 for 4 mo. at 6%?

6. How much should be paid to settle a loan of \$400 for 6 mo. at 6%?

7. If the interest at 6% on a loan is \$54, what is the interest at 5%?

8. If a loan of \$400 earns \$10 interest in 6 mo., what annual rate of interest does it earn?

9. A 30-day note, dated Sept. 4, is discounted Sept. 8. What is the term of discount?

10. Find the discount and the proceeds of a 60-da. non-interest-bearing note for \$1200, discounted for its full term at 6%.

11. A 30-day trade acceptance for \$800, dated Oct. 31, is discounted on Nov. 15 at 6%. Find the discount and the proceeds.

12. On an interest-bearing note dated Jan. 5, the face is \$720; the rate, 4%; and the term, 60 da. Find the amount due at maturity.

Marjorie received $100 at Christmas time. Instead of keeping the money at home, she decided to put it in a savings bank, where it would earn interest.

At the bank, Marjorie learned that the bank paid 2% interest annually on savings accounts, and that the interest was figured and paid on Jan. 1 and July 1 of each year. If she did not draw out the interest, it would be added to the principal and in turn would draw interest.

Interest paid on principal and interest is called *compound interest.*

How to Find Compound Interest

On Dec. 30, 1939, Marjorie deposited $100 in a savings account. Find the amount in her account on July 1, 1941, if the bank paid 2% interest, compounded semiannually, beginning Jan. 1, 1940.

$100.00 principal Jan. 1, 1940
1.00 interest for 6 mo.
$101.00 principal July 1, 1940
1.01 interest for 6 mo.
$102.01 principal Jan. 1, 1941
1.02 interest for 6 mo.
$103.03 principal July 1, 1941

Since the interest is 2% for 1 yr., it is 1% for 6 mo. Why?

Find the interest on the principal for the first period, and add it to the principal to get a new principal.

Find the interest on the new principal, and continue finding a new principal and interest until the final date, reckoning interest on dollars only.

Marjorie had $103.03 in her account on July 1, 1941.

Find the amounts of the following savings deposits:

1. $800 for 3 yr. at 3%, compounded semiannually.
2. $450 for $2\frac{1}{2}$ yr. at 2%, compounded quarterly.

Savings banks accept very small deposits. Sums which are deposited during the first three days of any month will begin to draw interest from the first of that month. Deposits made after the first three days of a month will draw interest beginning with the first day of the following month. Interest is figured on whole dollars only.

A check may be drawn against a savings account provided the passbook accompanies the check or is left at the bank until the check is paid and deducted from the account. Withdrawals made during any month are considered as made on the first of the month and do not draw interest for the month in which they are made.

Following is part of a page from Ted Ward's passbook.

Date	Withdrawals	Deposits	Interest	Balance
Jan 2 1940		300		300
Apr 12 1940		30		330
July 1 1940			3 10	333 10
Nov 15 1940	20			313 10
Jan 1 1941			3 27	316 37

Here is the way the interest was figured to July 1:

$300 drew interest from Jan. 1 to May 1, or for 4 mo.
$330 drew interest from May 1 to July 1, or for 2 mo.

Interest on $300 at 2% for 4 mo. = $2.00
Interest on $330 at 2% for 2 mo. = 1.10
Interest entered on July 1 = $3.10

Ted drew out $20 on November 15.

$333.10 drew interest from July 1 to Nov. 1, or for 4 mo.
$313.10 drew interest from Nov. 1 to Jan. 1, or for 2 mo.

1. Find the interest on the above balances (dollars only) at 2%, and check the amount of interest entered in the passbook as of Jan. 1, 1941.

Everyone should plan to save regularly, since regular savings—even when small—accumulate steadily. If regular savings are deposited in a savings account, they will draw compound interest. The table below shows how monthly deposits of \$1 to \$5 made regularly in a savings account will grow at 2% interest, compounded semiannually, over a period of 1 to 5 years.

HOW REGULAR DEPOSITS GROW IN A SAVINGS ACCOUNT AT 2% INTEREST COMPOUNDED SEMIANNUALLY

Monthly Deposits	1 Year	2 Years	3 Years	4 Years	5 Years
\$1	\$ 12.12	\$ 24.48	\$ 37.08	\$ 49.48	\$ 63.05
2	24.26	49.00	74.24	99.99	126.25
3	36.38	73.48	111.33	149.94	189.32
4	48.52	98.01	148.50	200.00	252.54
5	60.64	122.50	185.59	249.95	315.60

Use the table in solving these problems:

1. Tom Carter has a savings account in a bank which pays 2% interest, compounded semiannually. If he carries out his plan to save \$4 each month, how much will he have in his account at the end of 1 yr.? 3 yr.? 5 yr.?

2. Mary Wilkins can afford to save only \$1 each month. If she deposits this amount each month in a bank which pays 2% interest, compounded semiannually, how much will she have at the end of 2 yr.? 4 yr.?

3. John Paxton deposited \$3 each month in a savings account which drew 2% interest, compounded semiannually. How much had he in his account at the end of 5 yr.? What was the total amount of money he had deposited during the 5 yr.? How much interest had his money earned?

4. At 2%, compounded semiannually, how much would you have to deposit each month in order to have about \$250 at the end of 4 yr.?

Compound interest is often figured by the use of tables. Below is part of a compound interest table showing the amount of $1 at interest compounded annually at different rates for different periods. In this table, the interest has been figured on both dollars and cents.

TABLE SHOWING THE AMOUNT OF $1 AT COMPOUND INTEREST

Years or Periods	Rate of Interest, Compounded Annually						
	1%	1½%	2%	3%	4%	5%	6%
1	1.0100	1.0150	1.0200	1.0300	1.0400	1.0500	1.0600
2	1.0201	1.0302	1.0404	1.0609	1.0816	1.1025	1.1236
3	1.0303	1.0457	1.0612	1.0927	1.1248	1.1576	1.1910
4	1.0406	1.0613	1.0824	1.1255	1.1699	1.2155	1.2625
5	1.0510	1.0772	1.1041	1.1593	1.2167	1.2763	1.3382
6	1.0615	1.0934	1.1262	1.1941	1.2653	1.3401	1.4185
7	1.0721	1.1098	1.1487	1.2299	1.3159	1.4071	1.5036
8	1.0829	1.1265	1.1717	1.2668	1.3686	1.4775	1.5939
9	1.0937	1.1434	1.1951	1.3048	1.4233	1.5513	1.6895
10	1.1046	1.1605	1.2190	1.3439	1.4802	1.6289	1.7909
11	1.1157	1.1779	1.2434	1.3842	1.5395	1.7103	1.8983
12	1.1268	1.1956	1.2682	1.4258	1.6010	1.7959	2.0122
13	1.1381	1.2135	1.2936	1.4685	1.6651	1.8857	2.1329
14	1.1495	1.2317	1.3195	1.5126	1.7319	1.9800	2.2609
15	1.1610	1.2502	1.3459	1.5580	1.8009	2.0789	2.3966

To find the amount of $1 at compound interest for any period at a given rate, find the period in the first column and follow across the table to the amount given in the column under the given rate.

1. What is the amount of $1 in 7 yr. at 2% interest compounded annually; in 3 yr. at 1½%; in 10 yr. at 4%?

What is the amount of $650 in 6 yr. at 3% interest compounded annually? The amount of $1 in 6 yr. at 3% compound interest is $1.1941. $650 \times \$1.1941 = \$776.17$. The compound interest alone = $776.17 − $650 = $126.17.

2. Find the amount of $200 in 8 yr. at 5% interest compounded annually; of $475 in 12 yr. at 4% compounded annually.

You can use the table on the opposite page to find interest compounded semiannually.

An annual rate of interest of 4% is the same as 2% for each half year with twice as many interest periods.

Find the amount of \$1 for 5 yr. at 4% compounded semiannually. This is the same as \$1 for 10 interest periods at 2%, or \$1.2190. The amount of \$500 for 5 yr. at 4% compounded semiannually is 500 × \$1.2190, or \$609.50.

3. How would you find the amount of \$100 for 2 yr. at 6% compounded semiannually?

Use the table to find the amounts of the following:

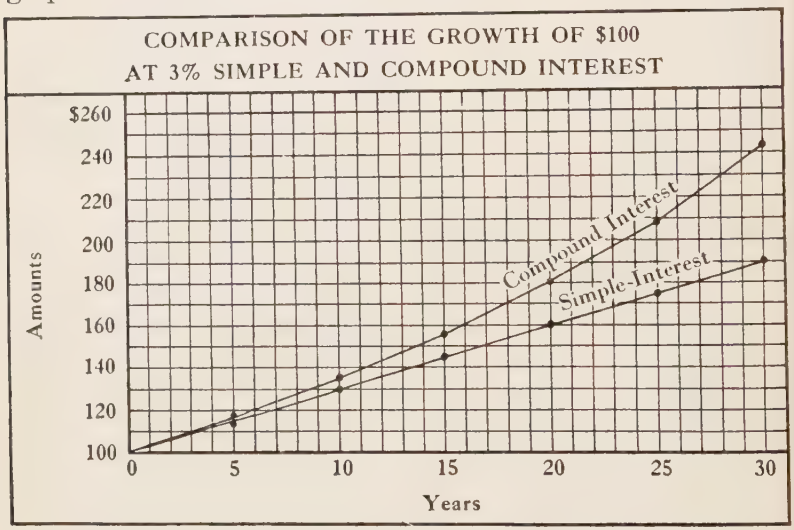
	Compounded Annually				Compounded Semiannually		
	Prin.	Time	Rate		Prin.	Time	Rate
4.	\$600	4 yr.	$1\frac{1}{2}\%$	**8.**	\$520	5 yr.	3%
5.	\$9000	3 yr.	2%	**9.**	\$2800	7 yr.	6%
6.	\$416	15 yr.	3%	**10.**	\$4500	2 yr.	2%
7.	\$12,000	11 yr.	4%	**11.**	\$750	6 yr.	4%

12. A man invested \$800 at 4% compounded semiannually. What was the amount of his investment at the end of $3\frac{1}{2}$ yr.?

13. Mr. Wicks kept \$2000 hidden under a floor board in his attic. How much would this sum have amounted to in 5 yr. at 2% compounded semiannually?

14. Mr. Shattuck invested \$3000 at 4% compounded semiannually. Find the amount for 7 yr. How much less would the investment have amounted to at 3% simple interest?

You have seen that money at compound interest grows faster than money drawing simple interest, provided the principals and the interest rates are the same. Study the graph below.



1. According to the graph, about what is the amount of \$100 at 3% simple interest for 10 yr.? At 3% compound interest for 10 yr.? Give the amounts at simple and at compound interest for 15 yr.; 20 yr.; 25 yr.; 30 yr.

2. Make a bar graph comparing the amounts of \$500 for 15 yr. at 3% simple interest and at 3% interest, compounded annually. Use the table on page 394 to help you in finding the amount at compound interest.

3. Find the difference in the amount of \$5000 invested for 5 yr. at 6% simple interest and the amount of the same sum invested for 5 yr. at 6%, compounded semiannually. Use the table on page 394 to find the amount at compound interest. What is the difference in the interest earned?

Work the examples in column A. If you fail on any example, do the example opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review
1. Finding the Bank Discount and the Proceeds of Non-Interest-Bearing Notes, Discounted for the Full Term		
Face Rate Term \$1200 6% 90 da. \$1655 $4\frac{1}{2}$% 5 mo.	Face Rate Term \$3000 6% 30 da. \$1270 $4\frac{1}{2}$% 4 mo.	384
2. Finding the Bank Discount and the Proceeds of Non-Interest-Bearing Notes, Discounted for a Partial Term		
Face Date Term \$720 May 12 60 da. Discounted May 21 @ 6% \$456 July 16 4 mo. Discounted Aug. 2 @ 5%	Face Date Term \$780 Oct. 12 90 da. Discounted Nov. 7 @ 6% \$388 Mar. 11 3 mo. Discounted Apr. 5 @ 5%	385
3. Finding the Bank Discount and the Proceeds of Interest-Bearing Notes		
Face Date Term % Int. \$648 Oct. 10 4 mo. 6% Discounted Oct. 10 @ 6% \$1500 Mar. 10 30 da. 5% Discounted Mar. 19 @ 6%	Face Date Term % Int. \$540 Jan. 5 3 mo. 6% Discounted Jan. 5 @ 6% \$2400 June 12 90 da. 4% Discounted June 30 @ 6%	386-387
4. Finding the Amount at Compound Interest		
Prin. Rate Time \$3000 4% 3 yr. Compounded annually \$6300 2% 2 yr. Compounded semiannually	Prin. Rate Time \$4000 3% 4 yr. Compounded annually \$8200 4% 2 yr. Compounded semiannually	391

Before doing the following exercises, members of the class should get information about registered mail rates and secure application forms for postal and telegraph money orders, a specimen express money order and, if possible, a specimen travelers' check.

1. Name four safe ways of sending money besides personal checks and bank drafts.

2. What is a postal money order? An express money order? Is it necessary to take a postal money order to the post office to have it cashed? Explain.

3. Using the rates printed on the back of the money order application blank, find the cost of money orders for: \$16; \$25.62; \$5; \$3.50; \$88.67; \$127.50.

4. What is meant by "registering" a letter? John enclosed \$40 cash in a letter to his sister. He registered the letter, valuing it at the amount of money enclosed. He asked for a "return receipt." Including 3¢ letter postage, what was the total cost of sending the letter?

5. What is a telegraph money order? Find the cost of a telegraph order for \$250 if the telegram costs 88¢ and the fee is 85¢ for the first \$100 and 25¢ for each additional \$100 or fraction thereof.

6. What is a bank draft? Allowing a fee of $\frac{1}{10}\%$, find the cost of bank drafts for: \$800; \$125.30; \$318; \$1560.

7. What is a travelers' check? Find the cost of twelve \$20 travelers' checks at a fee of $\frac{3}{4}\%$.

8. Describe some situations in which each of the following might be used in sending or carrying money: personal check, postal money order, express money order, telegraph money order, bank draft, registered mail, travelers' check.

Bound for Europe!

When Ruth and Jack Ellis went to Europe with their parents, they found they had to exchange United States money for the money of each country they visited. In England, they exchanged part of their money for **pounds, shillings,** and **pence** (plural of **penny**).

TABLE OF ENGLISH MONEY
1 penny (*d.*) = about 2¢ in U. S. money
12 pence (*d.*) = 1 shilling (*s.*) = about 25¢
20 shillings (*s.*) = 1 pound (£) = about $4.96

Although the value of the pound in United States money varies from time to time, it is normally worth about $5. The current exchange values of the pound and other foreign moneys can usually be found on the financial pages of the larger newspapers under the heading **Foreign Exchange.**

1. If 1 pound (£1) is worth $5, how many shillings could you get for $15?

2. In London, Ruth bought a coat for £2 15*s*. If £1 equaled $4.96, what was the cost in U. S. money?

*Optional material.

The Ellises found that, with the exception of England, each European country they visited had a standard money unit which, like our dollar, was divided into 100 parts.

STANDARD MONEY UNITS OF FOREIGN COUNTRIES

Country	Standard Money Unit		Normal U. S. Value
England	pound	= 20 shillings	$4.96
France	franc[1] (fr.)	= 100 centimes[2] (c.)	$ .03
Germany	mark (M.)	= 100 pfennigs[3] (pf.)	$ .40
Italy	lira[4] (l.)	= 100 centesimi[5] (c.)	$ .05
Netherlands	gulden[6] (g.)	= 100 cents (c.)	$ .55

1. frăngk. 2. sän'tēm. 3. pfĕn'ĭgz. 4. lē'rä. 5. sĕn-tĕs'ĭ-mē. 6. gōōl'dĕn.

Using the above table, find the approximate value in United States money of each of the following amounts:

1. 68 fr.
2. 142 M.
3. 215 l.
4. 456 g.
5. 120 fr. 40 c.
6. 216 M. 25 pf.
7. 86 l. 42 c.
8. 420 g. 50 c.

9. In Paris, Mrs. Ellis bought some lace for 400 fr. Find the cost of the lace in our money if 1 fr. = $.03.

10. In the Netherlands, Ruth bought a Dutch costume for $16\frac{1}{2}$ guldens. If 1 g. = $.55, how much did the costume cost her in United States money?

11. Mr. Ellis had a travelers' check for $20. If 1 lira = $.05, how many liras could he get for the check in Italy?

12. In Germany, Mr. Ellis paid a bill of 36 M. 40 pf. At a special rate of 23.15¢ per mark to travelers, how much was his bill in United States money?

13. Compare the value in United States money of fortunes of $1,000,000, 1,000,000 fr. (when 1 fr. = 2.78¢), and £1,000,000 (when £1 = $4.84).

*Optional material.

Explain how you would work each of the following examples; then work it:

1. Find the interest on \$730 for 30 da. at 6%.

2. Prove that $77.1 \div 1.5 = 51.4$

3. Find the common fraction equivalent of .1875.

4. Find the decimal equivalent of $\frac{9}{16}$ in.

5. Arrange in order of size from largest to smallest: $\frac{9}{40}$, .195, and 21%.

6. Divide $\frac{7}{24}$ by $2\frac{5}{8}$.

7. Using short cuts, first multiply and then divide 37,500 by $16\frac{2}{3}$.

8. Change 1360 lb. to a decimal of a ton.

9. Subtract 5 gal. 3 qt. from 8 gal. 1 qt.

10. Find what per cent 16 is of 25.

11. Find the difference between 20% of \$840 and 25% of \$840.

12. Find your per cent of profit based on the cost if you paid \$17.50 for a bicycle, \$2.50 for repairs, and sold the bicycle for \$24.

13. If $2\frac{1}{2}\%$ of a number equals \$50, what is 8% of the number?

14. Find the net proceeds if you sell 10 books at \$2.25 each and receive 20% commission.

15. A desk that cost \$12.50 was sold for \$18.75. Find the per cent of profit based on the selling price.

1. Write in figures: Four hundred twenty-seven and fifteen thousandths.

2. To which is $33\frac{1}{3}\%$ equal: $.033\frac{1}{3}$, $3.3\frac{1}{3}$, or $\frac{1}{3}$?

3. Express as decimals: 28%; $37\frac{1}{2}\%$; 3%; 75%; $\frac{1}{4}\%$.

4. Which of the following indicates what part 9 is of 54: $\frac{1}{9}$, 6, $.16\frac{2}{3}$, or .06?

5. Which is larger, .0025 or .005? How much larger?

6. 175 is what fractional part of 1000?

7. What is 25% of 356? $162\frac{1}{2}\%$ of 528?

8. Find the total of the following sales slips: \$34.57, \$26.42, \$8.88, \$.94, and \$52.64.

9. $\frac{5}{6}$ of $324 = ?$ $\frac{4}{5}$ of $\$17.25 = ?$ $\frac{7}{12}$ of $\$165 = ?$

10. From the sum of 5027 and 98,456, take 4527.

11. From 475.13, take 217.941.

12. From $62\frac{1}{3}$, subtract $27\frac{1}{6}$.

13. Find the sum of $3\frac{3}{4}$, $2\frac{7}{8}$, $4\frac{3}{8}$, and $3\frac{5}{16}$.

14. Express as a decimal the ratio of 132 to 160.

15. Multiply $4\frac{7}{8}$ by 24.

16. Multiply 56.019 by 2.93.

17. Divide 15,111 by 207, and check the answer.

18. Divide 347.27 by 57.4.

19. Divide $7\frac{2}{9}$ by $1\frac{5}{8}$.

20. $\frac{3}{4}$ of how many gallons = 153 gal.?

Give the meaning of each of these terms:

1. Interest
2. Principal
3. Rate of interest
4. Promissory note
5. Bank discount
6. Bank draft
7. Trade acceptance
8. Bank statement
9. Commission
10. Cash discount
11. Travelers' check
12. Deposit slip

Explain the difference between:

13. Exact interest and interest figured in the usual way.
14. Simple interest and compound interest.
15. A demand note and a time note.
16. A personal check and a bank draft.
17. A savings account and a checking account.
18. Endorsing a check for deposit and endorsing a check for transfer to someone else.
19. Bank discount and trade discount.
20. A date of maturity and a date of discount.
21. A trade acceptance and a promissory note.
22. The maker of a note and the payee.
23. A certified check and an ordinary check.
24. The endorser of a note and the maker of a note.
25. The term of a note and the term of discount.

Make and work four problems, each of which uses one of the four groups of terms below:

26. Principal, interest, yearly rate of interest
27. Promissory note, bank discount, proceeds
28. Non-interest-bearing note, date of discount, rate of discount, proceeds
29. Interest-bearing note, date of maturity, rate of interest, rate of discount, proceeds

Explain how you would find the answer in each of the following. Show you are right by making a problem with reasonable numbers and working and checking it.

1. A dealer holds a bargain sale offering a certain per cent off his regular prices. How can you find what you must pay him for a set of golf clubs?

2. How can Mr. Newcomb find the freight charge on a fraction of a ton of freight when the freight rate per hundred pounds is given?

3. A steamer trunk was sold at a discount. If you know what fractional part the sale price was of the regular price, how can you find what the regular price was?

4. Mrs. Simmons bought a length of velvet containing several yards and a fraction of a yard. If you know how much she paid for the entire piece, how can you find the price per yard?

5. Jack bought some young turkeys, cared for them for a few months, and sold them at a profit. How can he find his rate of profit based on the selling price?

6. Della is paid a weekly wage plus a commission on her sales. How can she find her earnings for a week?

7. You borrow money for garden tools and seeds. How can you find the amount you must pay back, including interest?

8. If you invest money at interest, compounded annually, how can you find the amount at the end of 2 yr.?

1. At what price must a store sell saucepan sets that cost 48¢ in order to allow 25% for overhead and 15% for profit, based on the selling price?

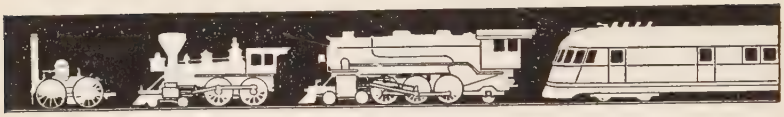
2. During a sale, a piano dealer offered pianos at 15% discount from the regular prices. Mrs. Fletcher bought one for $267.75. What was the regular price of the piano?

3. The United States exported 4,452,000 bbl. of wheat flour during a recent year. The previous year, 3,660,000 bbl. were exported. About what per cent did the exports of wheat flour increase?

4. Mr. Bates bought 600 bu. of potatoes for the Premier Market. The potatoes cost $.85 a bushel, the trucking charge was $2\frac{1}{2}$¢ a bushel, and Mr. Bates' commission was 4%. Find the total cost of the potatoes to the Premier Market.

5. Furniture listed at $2400 is needed for a new office. The Dallas Supply Company quotes a single discount of 45%; the Office Supply Company quotes successive discounts of 30% and 20%. Find the net cost in each case. How much would the buyer save by accepting the better of the two offers?

6. One year, the number of cattle in the United States was 68,279,000. In that year, the Central States had 31,779,000 cattle; the Southern States, 18,764,000; the Western States, 11,157,000; and the Eastern States, 6,579,000. About what per cent did each section have of the total number of cattle in the United States? Show these per cents in a bar graph.



1.
$$\begin{array}{r} 85569 \\ 7 \\ 878 \\ 49303 \\ 268 \\ 80 \\ \underline{7338} \end{array}$$

2. Subtract 275.345 from 374.224.

3. $4\frac{1}{2}\%$ of 280 = ?

4. 6 sq. ft. 34 sq. in.
$\times 6$

5. $39\frac{1}{8} - 14\frac{3}{4} = ?$

6. $4\frac{2}{3} \div 5\frac{1}{4} = ?$

7. 85 = ?% of 125

8. \$95 = 250% of ?

9. Multiply 359.2 by .087, and divide the product by 3.2.

10. On June 10, your checkbook shows a balance of \$212.19. On June 11, you deposit \$29.80 and draw check No. 53 to the Frazer Hat Shop for \$5.95. Write a check and stub to show these facts and the balance which will be left in your checking account.

11. On Feb. 15, you borrowed \$75, giving your note for 90 da. On that date, the bank discounted your note at 6%. What proceeds did you receive?

12. Find the amount due on a demand note for \$640, dated Apr. 19 of last year and paid June 4 of this year, with interest at 6%, if no interest has been paid.

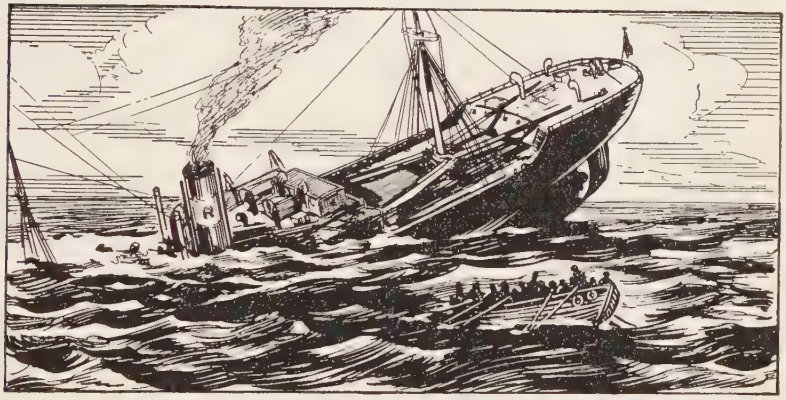
13. Using the 60-da. method, find the interest on a loan of \$580 for 3 mo. 18 da. at 5%.

14. A 90-da. non-interest-bearing note for \$998, dated Aug. 5, is discounted on Aug. 9 at 6%. Find the bank discount and the proceeds.

15. On June 29, Mr. Turner discounts a 90-da. note for \$256, dated June 14, and bearing 6% interest. Find the proceeds if the note is discounted at 6%.

*Record your score on your Progress Chart. See page 321.

UNIT IV. INSURANCE: PROTECTION AGAINST LOSS



To the bottom of the sea. Ship and cargo lost!

In days gone by, when a ship sank at sea, a house burned down, or a flood swept away a farmer's barns and stock, the owner had to stand the loss. The need for protection against such losses gave rise to **insurance.**

The way insurance works can be illustrated by fire insurance. Anyone who wishes to protect himself from loss by fire pays a fire insurance company a small sum of money. The many small sums paid the insurance company make a large sum from which are paid the losses of those who have fires. Those who do not have fires help to pay the losses of those who do, but they themselves are also protected.

Today, one can get insurance which covers loss from almost any cause. These causes include fire, theft, destruction and damage to property, injuries done to others, accidents, death, illness, unemployment, old age, and many others.

Fire under control — in stoves, in locomotives and boiler rooms, and in the great furnaces in which ores are smelted to make iron and steel—is a friend that makes possible many of the things we enjoy today. But fire out of control is a dangerous foe that causes tremendous loss of life and property.

For a small sum of money, fire insurance companies will insure a person against loss from damage to his property due directly or indirectly to fire.

When Mr. Dodge took out fire insurance on his home, the insurance company gave him a **policy,** or written agreement. The policy gave: (1) Mr. Dodge's name as **the insured;** (2) a description of the property; (3) the **face** of the policy, or amount for which the property was insured; (4) the **premium,** or charge for insurance; (5) the **term,** or length of time the insurance was to be in force.

The policy also contained the conditions as to payment of losses. If Mr. Dodge's house was completely burned, the insurance company would pay him its value up to the amount of the policy. If the house was only partly damaged due to fire, the company would pay him the cost of repairs.

Rates for fire insurance on different buildings vary according to the risk, or probability of fire. The greater the risk, the higher the rate. In a city where there is ample fire protection and water supply, rates are lower than in the country. The rate on a building built of brick or any fire-resisting material is less than on a wooden building. Why?

Fire insurance rates are usually given as a certain charge per \$100 of insurance or as a per cent of the amount of insurance—the face of the policy.

How to Find the Fire Insurance Premium When the Rate Is Given

Find the fire insurance premium charged for insuring Mr. Dodge's house for \$6000: (1) at a yearly rate of 48¢ per \$100; (2) at a yearly rate of $\frac{1}{2}\%$ of the face of the policy.

1.

\$6000 = 60 hundred dollars

\$.48 × 60 = \$28.80 premium

Multiply the rate per \$100 by the number of hundreds of dollars in the face of the policy.

2.

\$6000 × .00$\frac{1}{2}$ = \$30 premium

Multiply the face of the policy by the rate per cent.

Fire insurance is usually issued for one, three, or five-year terms. The three-year rate is $2\frac{1}{2}$ times the one-year rate. The five-year rate is 4 times the one-year rate. If the one-year rate is 45¢, the three-year rate is \$1.125 and the five-year rate is \$1.80. If the one-year rate is $\frac{1}{2}\%$, the three-year rate is $1\frac{1}{4}\%$ and the five-year rate is 2%.

Find the premium on each of the following policies for a term of: (a) one year; (b) three years; (c) five years:

	Face	Yearly Rate per \$100		Face	Yearly Rate per \$100		Face	Yearly Rate Per Cent
1.	\$1500	75¢	5.	\$21,000	\$ .75	9.	\$18,000	$\frac{3}{4}\%$
2.	\$2600	34¢	6.	\$275,000	\$ .21	10.	\$32,000	$1\frac{1}{4}\%$
3.	\$4800	54¢	7.	\$46,300	\$1.55	11.	\$125,000	$\frac{1}{4}\%$
4.	\$6750	48¢	8.	\$38,500	\$1.25	12.	\$243,000	$1\frac{1}{2}\%$

Fire insurance on a building does not cover the contents of the building. Separate insurance must be taken out on household goods, merchandise, and other contents.

Because most fires cause only partial damage, owners often have their buildings and the contents insured for less than their full value. A man may have his house insured for $\frac{3}{4}$ of its value of \$12,000, which is \$9000. If the house is totally destroyed by fire, he will receive only \$9000, the face of the policy. If the house is partly destroyed, the insurance company will pay for the damage up to the value of \$9000.

Remember: A fire insurance company is not liable for more than the amount of actual loss up to the face of the policy.

1. Mr. Claxton insured a building against fire for $\frac{3}{4}$ of its value of \$20,000. A fire resulted in damage of \$18,650. How much did the insurance company pay? Why? How much was Mr. Claxton's loss?

2. Dr. Martin insured his \$14,000 home in the city for 80% of its value. The term of his policy was 5 yr., and the yearly rate was 35¢ per \$100. How much was his premium?

3. Dr. Martin also took out a 3-yr. fire insurance policy for \$1500 on his summer camp, at a yearly rate of 85¢ per \$100. Find the premium. Why was the rate higher for Dr. Martin's summer camp than for his city house?

4. Mr. Kennedy had \$12,000 worth of goods in his store. He took out fire insurance on the stock for 1 yr. for $\frac{2}{3}$ of its value. (a) What was the face of his policy? (b) At a yearly rate of 65¢ per \$100, how much was the premium? (c) A fire occurred in Mr. Kennedy's store, and the insurance company figured the loss at \$6743. Did the company pay the entire loss? Why? If the loss had been \$9850, what amount would the insurance company have paid Mr. Kennedy?

5. A store is insured for \$75,000, and its contents are insured for \$125,000. The yearly rate on the building is 21¢ per \$100; and on the contents, \$.45 per \$100. What is the total cost per year for insurance?

6. Mr. Barr insures his household furniture for \$3000 at a yearly rate of 32¢ per \$100. He renews the insurance each year. How much less would the insurance cost him in 5 yr. if he took out a 5-yr. policy instead of paying yearly premiums?

7. Every 5 yr., Mr. Harriman renewed his fire insurance policy for \$25,000 on his store. The rate for each 5-yr. term was \$1.60 per \$100. Twenty-two years after the insurance was first taken out, the insurance company paid him a fire loss of \$4972. How much more was this than Mr. Harriman had paid in premiums?

8. Mr. Langley had been paying a premium of $1\frac{3}{4}\%$ per year for a \$40,000 fire insurance policy on his factory. After he had an automatic sprinkler system installed, the insurance company reduced his premium rate to 60¢ per \$100. How much was his yearly saving in premiums? The sprinkler system cost \$2300. In how many years did the savings in premiums pay for the sprinkler system?

9. Mr. Tilden bought a house and lot for \$9000. Mrs. Tilden wished to have the property insured for its full value. Mr. Tilden, however, explained that the land, which was worth \$1250, would not burn. If he subtracted the value of the land from the total cost of the property and took out fire insurance for the balance:

a. What was the face of the policy?

b. What was his premium for a 3-yr. policy if the yearly rate was 28¢ per \$100?

c. How much less did he pay in premiums by not taking out a 3-yr. policy for the full value of the property?

In a recent year, 40,300 persons were killed and 1,221,090 were injured in automobile accidents in the United States.

An automobile is the source of much pleasure and is a most convenient means of getting about, but it may easily cause loss, injury, or death. Every year, thousands of persons are killed by automobiles, many more thousands are injured, and motor accidents result in an untold amount of damage to cars and to other property.

Few people could afford to take the financial risks that come with driving a car if it were not for automobile insurance. Every automobile owner is held liable, or responsible, by law for any injuries caused by his car to other persons or to their property if the driver is proved to be at fault. Insurance companies issue different kinds of insurance to protect a car owner against losses from claims for such damages. They will also insure him against loss if his own car or its equipment is burned, damaged, or stolen.

Public liability insurance protects the car owner from having to pay damages up to the limits of his policy if his car kills or injures anyone. A public liability policy for \$10,000-\$20,000 means the insurance company will pay as high as \$10,000 if one person is injured or killed, or as high as \$20,000 if two or more persons are injured or killed in one accident.

A public liability policy may be for as low as $5000-$10,000, but the rate charged for greater protection is so little more that it is well worth while to carry at least $10,000-$20,000 public liability if possible.

Property damage insurance protects the car owner up to the limits of his policy against loss because of damage done by his automobile to another person's property.

Fire and theft insurance. These are frequently written together in one policy which protects the car owner against loss from damage to his car caused by fire, lightning, or explosion, and against loss from theft of the car or its equipment in case it is stolen and not recovered.

In paying fire and theft losses, an insurance company takes into account the fact that cars and their equipment **depreciate,** or lessen in value. If a fully insured car is completely destroyed by fire or is stolen and not recovered, the insurance company will pay the actual cash value of the car and its equipment at the time the car was burned or stolen. If the fire damage is only partial, the insurance company will pay what it would cost to repair the damage. The amount of any loss is decided by claim adjusters of the insurance company.

Many fire and theft policies also include insurance against loss due to broken glass, damage from falling trees, floods, windstorms, riots, and other causes.

Collision insurance protects the car owner against loss from damage to his own car from collision. There are two types of collision insurance. Under one, the company agrees to pay the entire cost of necessary repairs. Under the other, the owner pays the first $25, $50, or $100, and the company pays the rest. The latter type has the lower rate.

The rates for public liability and property damage in-

surance vary throughout the country. In general, the rates for each city or section are based on the number and frequency of accidents that occur there. Rates are high in localities in which there are many accidents, and are low in places where there are few. The rates in some states depend upon the accident records of those who drive the car; the ages of the drivers—drivers under 25 paying a higher rate than those over 25; the number of people who drive the car; and whether the car is used for business or merely for pleasure driving. The rate for theft insurance is higher for light, inexpensive cars than for the more expensive makes, because the light cars are much more numerous and so are more difficult to trace, if stolen.

Six months ago, Mr. Trafton decided to buy a car and to take out public liability, property damage, and fire and theft insurance on it. First, he shopped for both new and used, low and high-priced cars. Then he asked about the annual insurance rates on those cars which interested him, saying he would be the only driver and the car would not be used for business. Next, he made out the following table.

	Make of Car	Price of Car	Rates per Year		
			Public Liability $5000-$10,000	Property Damage $5000	Fire and Theft per $100
1.	A, new	$750	$36.35	$9.25	$ .95
2.	A, used	$375	36.35	9.25	1.15
3.	B, new	$960	36.35	9.25	.80
4.	C, used	$950	36.35	9.25	.85
5.	D, new	$1500	36.35	9.25	.65

The lowest, or **minimum,** charge that many insurance companies make for fire and theft insurance is $5. Using the above table, find the total annual insurance Mr. Trafton would have to pay on each car listed if it were insured for its full value. The charge for fire and theft insurance should not be less than the $5 minimum rate.

6. Mr. Trafton bought car B. At the same time, he took out a public liability policy for \$10,000-\$20,000. The premium for this policy was only 15% more than for a \$5000-\$10,000 policy. (a) How much did the \$10,000-\$20,000 policy cost? (b) If Mr. Trafton's car killed a man and Mr. Trafton were found liable for \$9000, how much better off would he be with the \$10,000-\$20,000 policy than with a \$5000-\$10,000 policy? Disregard premiums.

7. When Mr. Trafton had had his car ten months, a spare tire that cost \$14.00 and a pair of fog lights worth \$8.00 were stolen. The insurance company figured the tire at 75% of its cost and the fog lights at full value. How much did the insurance company pay Mr. Trafton?

8. By paying a premium of \$28, Mr. Trafton could have had collision insurance covering damages to his car over \$50. His car skidded into a fence, and he had to pay \$175 for repairs to his car. Considering the premium, how much would Mr. Trafton have been ahead if he had carried collision insurance?

SOME QUESTIONS ABOUT AUTOMOBILE INSURANCE

1. In your opinion, should the rates for automobile insurance in your locality be high or low? Why? Find out what the rates are.

2. Do you consider it fair for a man who has very little money to drive a car without having public liability and property damage insurance? Explain.

3. Is a man justified in letting his liability and property damage insurance lapse, or run out, when his car becomes old? Why?

4. Is it real or false economy to drive a car, even if fully insured, after the tires have become old and smooth? Discuss.

1. Change to per cents: .07, .96, .005, 2.6, 1.08.

2. Change to decimals: 3%, 29%, $6\frac{1}{4}\%$, 166%, $\frac{4}{5}\%$.

3. Change to fractions or mixed numbers: 60%, 38%, 145%, $11\frac{1}{9}\%$, $2\frac{1}{2}\%$.

4. Change to per cents: $\frac{5}{6}$, $\frac{7}{20}$, $\frac{3}{16}$, $\frac{13}{40}$, $\frac{11}{12}$.

Find:

5. 25% of 640
6. $66\frac{2}{3}\%$ of 111
7. $162\frac{1}{2}\%$ of 160
8. 3% of 1200
9. 8% of 140
10. 28% of 850
11. $\frac{3}{4}\%$ of \$148
12. $\frac{2}{5}\%$ of 2500 bu.
13. $1\frac{1}{3}\%$ of 180 gal.

Find what per cent the first number is of the second:

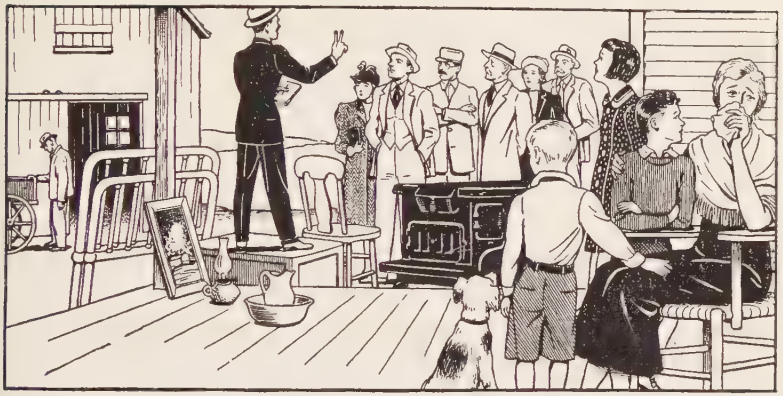
14. 36, 48
15. 120, 80
16. 48, 16
17. \$24, \$600
18. \$37, \$400
19. \$88, \$500
20. 117 lb., 650 lb.
21. 2.14 ft., 428 ft.
22. 224 mi., 6400 mi.

Find the number of which:

23. 16 = 25%
24. 25 = $33\frac{1}{3}\%$
25. 120 = 75%
26. 27 = 9%
27. 124 = 31%
28. 750 = 250%
29. 86 = $\frac{1}{2}\%$
30. 12.6 = 3%
31. 90 = $4\frac{1}{2}\%$

Find the interest on:

	Prin.	Rate	Time		Prin.	Rate	Time
32.	\$424	6%	60 da.	36.	\$714	6%	84 da.
33.	\$912	5%	30 da.	37.	\$1800	5%	2 mo. 15 da.
34.	\$448	6%	36 da.	38.	\$3680	3%	45 da.
35.	\$1240	$4\frac{1}{2}\%$	90 da.	39.	\$172	6%	5 mo. 6 da.



Because the father carried no life insurance to provide for his family, home and belongings must be sold.

Dick Evans, 25 years old and married, was whistling as he stepped off the curb to cross the street. Brakes screeched, a car swerved, and Dick leapt back. No harm done; but as Dick went on, he said to himself, "That was a close call! What would have happened to Mary if I had been killed? She would have practically no money to live on. I'd better get some life insurance right away."

That afternoon, Dick called on Mr. Gray, an insurance man, and told him he wanted to learn about life insurance and what it would do for him.

MR. GRAY: "You can take out life insurance simply as protection for your family in case of your death, or you can combine protection with savings."

DICK: "I want first of all to provide for my wife. But what are the different kinds of policies I might take out?"

MR. GRAY: "Study this card. It will tell you about the most common types of life insurance policies and the advantages of each."

FOUR COMMON TYPES OF LIFE INSURANCE

Ordinary Life	Limited-Payment Life	Endowment	Term
On the death of the insured, the face value of the policy is paid to the beneficiary—the person for whose benefit the policy was taken out. Premiums are payable during the entire lifetime of the insured. Enables a person to provide for his beneficiary at low premium.	On the death of the insured, the face value of the policy is paid to the beneficiary. Payment of premiums limited to a definite term, such as 20 yr., after which policy becomes a **paid-up** policy, the insurance continuing until death of the insured. Provides for beneficiary at premium rates higher than for ordinary life, but payments limited to a specified number of years.	At the end of a definite period, such as 20 yr., the face value of the policy is paid to the insured. If he dies before the end of the period, the face value is paid to the beneficiary. Premiums payable during term of policy or until death of the insured before the term ends. Provides a plan for saving with protection for beneficiary during term of policy. Premium rates higher than for limited-payment life.	Provides protection for short periods. If the insured dies within the period, the face value of the policy is paid the beneficiary. At the end of the period, the policy is canceled. Premiums payable during term of the policy or until death of insured before the period ends. Provides temporary protection for beneficiary, at lowest premium.

Mr. Gray: "Before you can take out any of these policies, you must undergo a thorough medical examination by a doctor named by the insurance company. No insurance company can afford to insure a person who is not in good health."

Dick: "That's all right; but what about the amount of insurance I should take out and the premiums I must pay?"

Mr. Gray: "No one should take out more insurance than he can surely pay for, although all the policies described, except the term policy, have a **cash surrender value** to the insured after they have run two years. That means that if you can't pay the premiums or if for some reason you need

ready money, you can, after the policy is two years old, surrender it to the company and be paid cash for it. As the amount of cash you receive is based on the amount you have paid in premiums, the longer you have held the policy the greater the cash value will be.

"If you should be pressed for money and still not want to surrender your policy, the company will lend you the cash value of your policy, charging you interest on the loan. This we call the **loan value** of the policy.

"Another thing. If you find that you are unable to continue paying your premiums but wish to be covered by insurance, you may exchange your policy for a policy which gives you **paid-up insurance** in a smaller amount, for which no premiums are charged.

"Now, as to the cost of the different policies; study this table, which shows the yearly rate we charge for each $1000 worth of insurance at given ages."

YEARLY LIFE INSURANCE RATES PER $1000

Age When Insured	10-Year Term	Ordinary Life	20-Payment Life	20-Year Endowment
20	$10.40	$19.21	$29.39	$48.48
25	10.85	21.49	31.83	49.33
30	11.52	24.38	34.76	50.43
35	12.56	28.11	38.34	51.91
40	14.31	33.01	42.79	54.06
45	17.52	39.55	48.52	57.34
50	23.33	48.48	56.17	62.55
55	33.15	60.72	66.69	70.81

DICK: "According to the table, since I am 25, if I take out a $5000 ordinary life policy, it will cost me 5 × $21.49 or $107.45 a year. As long as I hold the policy, I must pay that amount; but if I were to die next month, you would pay my wife $5000."

Mr. Gray: "Right. On your death, we will pay her that amount in one sum or you can arrange for us to pay her, say, $1000 a year for five years or $500 a year for ten years, plus the interest that the balances earn; or we will pay her in monthly payments if that seems best to you. Sometimes, the gradual payment is a good idea, especially if the beneficiary is not accustomed to caring for large sums of money.

"There is one more thing you should know. For a slight additional premium, an **accident clause** or a **waiver clause** may be added to your policy. Under the accident clause, double the face of the policy is paid in case of death by accident. Under the waiver clause, the insured does not have to continue paying premiums if he becomes totally disabled."

Dick: "Could I pay my premium in quarterly or semi-annual payments?"

Mr. Gray: "Yes, but the rate is less if the premium is paid in one annual payment."

1. Dick decided on an ordinary life policy for $7000. What was his annual premium?

2. Using the table of rates on page 419, find the annual premium on each of the following policies:

	Face	Age	Type of Policy
a.	$6000	40	Ordinary life
b.	$4500	45	10-year term
c.	$8000	30	Ordinary life
d.	$12,000	25	20-payment life
e.	$3500	25	20-year endowment
f.	$2750	30	Ordinary life
g.	$16,500	35	20-year endowment
h.	$15,750	40	20-payment life

1. A man takes out a 20-year endowment life insurance policy for $5000 at an annual premium of $48.79 per $1000. If he lives the full term, how much more does he receive than he paid in premiums?

2. John Baxter took out a 20-payment life policy for $3000 when he was 18 yr. old. The annual rate was $28.53 per $1000, and the quarterly rate was $7.56. John decided to pay quarterly. What was his quarterly premium? How much more than the annual premium did the quarterly premiums cost him in a year?

3. When Mr. Taylor was 25 yr. old, he took out an ordinary life policy for $2500 on which he paid an annual premium of $21.49 per thousand. After paying 8 annual premiums, he died; and the company paid his beneficiary $2500. How much more did the beneficiary receive than Mr. Taylor had paid in premiums?

4. Mr. Davidson, who is 30 yr. old, has a wife and 3 young children. He feels that he can afford to spend $300 a year for insurance. At the rates given in the table on page 419, about how much insurance, to the nearest $100, can he have if he takes out a 20-payment life policy? An ordinary life policy?

5. Mr. Gates has an ordinary life policy for $3000. The cash, loan, and paid-up values for each $1000 of insurance as printed in his policy are given in the table below. What would be the cash or loan value of Mr. Gates' policy at the end of 5 yr.? The paid-up value?

End of	Cash or Loan Value per $1000	Paid-Up Ins. per $1000
5 yr.	$ 48	$111
10 yr.	116	244
15 yr.	198	378
20 yr.	296	514
25 yr.	388	614
30 yr.	483	701

6. Find the cash, loan, and paid-up values at the end of 10 yr., if Mr. Gates' policy had been for $5000. For $12,000.

Perhaps you've wondered how an insurance company arrives at the rates it charges.

First, the rates must be high enough to cover the probable losses during any year. Insurance companies have a record of what the annual losses have been for nearly every kind of risk over a long period of years. From these figures, they are able to estimate the amount of yearly loss they may be expected to meet. The premium rates cover the estimated losses.

Second, the rates must pay the company's overhead expenses, such as heat, light, rent, salaries, commissions.

Third, the insurance company must set aside reserve funds sufficient to take care of any unusual losses such as big city fires or epidemics. The insurance rates must provide for these funds. Since the reserve funds are carefully invested, they increase constantly.

Fourth, if people have money invested in the company, they should be paid a return on their investment. The insurance rates include a profit to allow for this.

When the total cost of doing business is less than was expected, many insurance companies return part of the money to their policyholders in the form of **dividends.**

1. Name some factors which would tend to reduce rates for fire insurance; life insurance; automobile insurance.

2. Find out why an insurance company charges a person the same rate each year on his life insurance policy although the older he gets the greater the risk to the company.

3. According to recent figures, Americans carry about $110,000,000,000 in life insurance alone, and the reserves held by the insurance companies amount to 28 billion dollars. Find out how this money is protected by law.

When young, we are apt to give little thought to the future, but as we grow older we begin to think about the years ahead. Will we always be able to earn a living, or will we be dependent upon others in our later years?

Through an **annuity,** a type of policy which guarantees the policyholder an annual income beginning at an agreed age, insurance companies enable us to provide for the future. Annuities may be had in many forms, the most common of which are:

Retirement Annuity. This type of annuity guarantees the policyholder a life income beginning at any selected age between 50 and 70. The premiums are based on the age of the insured at the time the policy is taken out, the retirement age, and the income desired. Premiums may be paid quarterly, semiannually, or annually until retirement age. If the insured dies before income payments have been made to him for ten full years, the remaining income is paid to his heirs over a period of years. Otherwise, the insured receives the income until his death.

Other types of annuity policies provide for a life income starting at any age from 20 on, and may be purchased outright by the payment of one large premium. The charges for these annuities depend upon the age of the person and the amount of income he desires or the amount of money he wants to invest in the annuity. The most common forms of this type of annuity are:

Life Annuity. Under this form, the insured receives a specified income for life, the contract ending with the last income payment made before his death.

Refund Annuity. This form also provides a stated income for life. If, however, the insured dies before the sum of the income payments made to him equals the amount of the premium he has paid, the income will continue to be paid

to his beneficiary until the income paid equals the premium paid. If the insured dies and the total income payments made to him equal the premium he has paid, the contract ends with his death.

Joint and Survivor. Under this plan, the income is paid during the joint lifetime of two people, as husband and wife. If one dies, the income is paid to the other, at whose death the contract ceases.

Special Life Annuity. This form provides a life income to the insured and a death benefit to the beneficiary upon the death of the insured.

1. Mr. Eaton is in the advertising business, but his hobby is raising hunting dogs. He plans to retire when he is 60 years old and follow his hobby.

To insure himself and his wife a regular income on retirement, Mr. Eaton, at the age of 40, took out a retirement annuity policy for which he pays an annual premium of \$600. If he will receive a monthly income of \$14.24 for each \$100 he pays in yearly premiums, what will his yearly income be?

2. Mr. Doyle at 50 took out a life annuity policy for which he paid a single premium of \$2238.78 for each \$10 of monthly income he receives. If his monthly income is \$150, how much did he pay for his policy?

3. For every \$1000 he paid for a refund annuity policy, Mr. Willis receives a monthly income of \$4.82. The cost of his policy was \$25,000. What is his yearly income?

One of the problems which every community has to face is the care of its old, needy, and dependent people and its people who are unemployed.

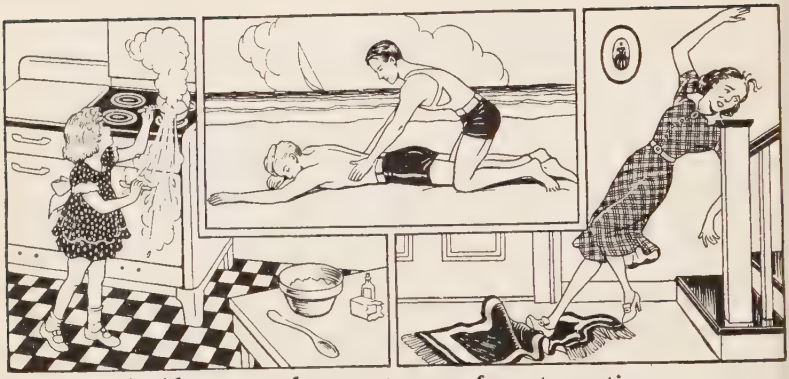
In 1935, the United States government passed the **Social Security Act** providing old-age and unemployment insurance for workers in business and industrial pursuits with the exception of agricultural and domestic help, government and railroad employees, and certain other classes.

Under the **old-age provision of the Act,** an employer is taxed 1% of each employee's wages up to $3000 for a given number of years, and the employee is taxed the same amount. The tax gradually increases until it is 3%, and remains at that figure. If a worker retires from active employment at the age of 65 years, he receives a monthly benefit ranging from $10 to $85, depending upon what his average monthly wage has been. If he has unmarried dependent children under 18 years of age, they each receive a monthly benefit equal to $\frac{1}{2}$ the worker's benefit; and his wife, if dependent, receives the same after she is 65.

The law also provides for the continuation of certain benefits to the family after the worker dies, or for the payment of a lump-sum benefit to his heirs if he leaves no wife or dependent children.

The **unemployment provision of the Social Security Act** provides for co-operation between the states and the federal government in the establishment of state unemployment insurance laws. Under the state laws, employers are taxed a certain per cent of their payrolls, and this money is used by the states to provide incomes for people out of work.

Besides the provisions for old-age and unemployment benefits, the Social Security Act also provides for aid to the states in caring for their needy aged and crippled people and dependent children.



Accidents may happen to any of us at any time.

1. In a recent year, accidents in our country caused an average of 12 deaths for every hour of the day. How many is that in one day? In a year of 365 da.?

2. In the same year, the number of persons injured in accidents averaged 190,000 a week. How many persons were injured in a year of 52 weeks?

3. It is estimated that the loss due to deaths, personal injuries, and damage to property caused by accidents amounts to an average of $10,000,000 a day. At that rate, what is the loss in a year of 365 da.?

4. In a recent year, the construction industries and the steam railways in the United States employed about the same number of workers. During the year, there were 2900 accidental deaths among the construction workers, and 750 among the railway workers. About what per cent more were the fatal injuries to construction workers than to railway workers?

5. While "accidents will happen," a large part of them can be avoided if care is used. Name some common accidents that happen in the home, in school, and on the streets; and tell ways to avoid them.

Injuries and illness are serious matters to a person who is dependent on his earnings. If a mechanic breaks his arm or a wage earner of any sort has a long illness, how is he to manage with his pay cut off and doctors' bills to meet? Again, the answer is insurance, this time **accident and health insurance.**

While there are many kinds of accident and health insurance policies, the following illustrates how this type of insurance operates.

To protect himself and his family, Mr. Ogden pays $82.50 a year for an accident and health policy under the terms of which:

a. His family will be paid $5000 in case of his death by accident.

b. He will be paid: (1) $5000 if, due to an accident, he is entirely disabled by such a disaster as the loss of both hands, both feet, or both eyes; (2) $2500 if he is partially disabled by the loss of one hand or foot or eye.

c. If he should be unable to work due to accident or illness, he will be paid, after the first two weeks, $25 each week he is ill up to 50 weeks.

d. The insurance company will also pay for doctors, nurses, and hospital care up to $1000 in case Mr. Ogden requires medical attention as the result of an accident or must have a surgical operation.

1. During the year in which Mr. Ogden took out his policy, he had to have his appendix removed. The surgeon charged $100, the hospital bill was $40, and special nurses charged $30. All this was paid by the insurance company. In addition, Mr. Ogden was unable to work for four weeks. How much did the insurance company pay?

2. In that same year, Mr. Ogden slipped on an icy pavement and broke his ankle. He was unable to work for 8 weeks. His medical expenses were $75. How much did his accident cost the insurance company?

3. Although Mr. Ogden proved a poor risk for the company that year, his insurance was continued the next year. Why could the insurance company continue to insure him?

4. Have some member of the class find out about two or three kinds of accident and health insurance policies commonly issued, and report his findings to the class.

5. Find out about **hospitalization insurance,** and discuss its advantages.

WORKMEN'S COMPENSATION

Many states have passed **Workmen's Compensation laws** which require employers to carry **employers' liability insurance** to protect their employees against loss due to injury or death resulting from their employment. In case a workman is injured while at work, any medical expenses arising from his injury are paid. After a given period, he is also paid a per cent of his wages for any additional time he is disabled. If the workman is killed, his dependents receive an amount which depends upon the circumstances of his death.

1. Under a Workmen's Compensation law, when James Brown broke his arm while at work his medical bill of $24.75 and two thirds of his weekly wage of $24 for nine weeks were paid. How much did the insurance company pay?

2. Learn what you can about the Workmen's Compensation law in your state, and report to the class.

In states which do not have Workmen's Compensation laws, employers generally protect themselves by taking out employers' liability insurance.

Today, insurance companies will issue insurance against loss from almost any kind of reasonable risk. Some of the more important special kinds of insurance are:

Business Insurance. Different types of insurance protect a business against losses from bad debts, claims for accidents or injuries to employees, the dishonesty of employees, and various forms of robbery and burglary. Some businesses also take out insurance against losses resulting from the death or disability of a partner or an important official, and from interruptions to business due to strikes and other disturbances.

Windstorm Insurance. Hurricanes, tornadoes, cyclones, and other windstorms often cause tremendous property damage. Insurance is available for protection against loss resulting from such storms.

Agricultural Insurance. Farmers and owners of businesses which depend upon agriculture often insure against loss from damage to crops from hail, frost, and other weather conditions.

Transportation Insurance. Types of transportation insurance are motor, rail, and aviation insurance, and marine insurance, which protects owners or shippers against loss of or damage to ships and cargoes from perils of the sea.

Industrial Life Insurance. People with very limited incomes can buy standard forms of life insurance by paying small weekly premiums to a collector. The face of the average industrial policy is small, and the cost is high because of the expense of weekly collections.

Other types of insurance cover bank deposits, parcel post shipments, and damage from flood.

1. Choose different members of the class to look up information about each type of insurance named above.

Work the examples in column A. If you fail on any example, do the example opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review
1. Finding the Premium on a Fire Insurance Policy		
Face / Yearly Rate / Term \$6000 / 48¢ per \$100 / 3 yr. \$32,000 / $1\frac{3}{4}\%$ / 1 yr. Value of property, \$12,800 Insured for $\frac{7}{8}$ value at \$1.22 per \$100 for 1 yr.	Face / Yearly Rate / Term \$8000 / 36¢ per \$100 / 3 yr. \$27,500 / $2\frac{1}{2}\%$ / 1 yr. Value of property, \$15,000 Insured for $\frac{5}{6}$ value at \$1.35 per \$100 for 1 yr.	409
2. Finding the Cost of Automobile Insurance		
Car insured for \$866 Annual insurance rates: Fire & Theft, 95¢ per \$100 Public Liability, \$38.50 Property Damage, \$10.15	Car insured for \$1098 Annual insurance rates: Fire & Theft, 80¢ per \$100 Public Liability, \$42.84 Property Damage, \$9.75	414
3. Finding the Premium on a Life Insurance Policy		
Face / Rate \$7500 / \$62.06 per \$1000 \$6200 / \$54.37 per \$1000	Face / Rate \$8500 / \$50.92 per \$1000 \$12,000 / \$30.31 per \$1000	419-420
4. Finding Annuity Incomes and Premiums		
Annual premium, \$400 Monthly income on retirement, \$19.32 per \$100 of annual premium paid Monthly income, ?	Annual premium, \$500 Monthly income on retirement, \$25.21 per \$100 of annual premium paid Monthly income, ?	423-424
Quarterly income, \$400 Single premium, \$1500.90 per \$25 of quarterly income Cost of policy, ?	Monthly income, \$100 Single premium, \$1288.98 per \$10 of monthly income Cost of policy, ?	

1. Define each of these terms as applied to insurance: policy, premium, premium rate, face of policy, risk.

2. Tell what each of the following kinds of automobile insurance covers: fire, theft, public liability, property damage, collision.

3. Give some of the advantages of carrying life insurance.

4. The older a person is when he takes out life insurance, the higher the premium he has to pay. Explain.

5. How does an ordinary life policy differ from a term policy? A limited-payment life from an endowment policy?

6. Explain what is meant by the cash surrender, loan, and paid-up insurance values of a life insurance policy.

7. What is an accident clause in a life insurance policy? A waiver clause?

8. Explain the difference between these different kinds of annuity policies: life annuity, refund annuity, joint and survivor annuity, special life annuity, retirement annuity.

9. Explain how old-age insurance is provided for under the Social Security Act. How does it differ from unemployment insurance?

10. Of what value is accident and health insurance?

11. What are Workmen's Compensation laws?

12. Name several special kinds of insurance which are taken out by business, commercial, and industrial firms.

13. Give reasons why you think insurance is a good investment.

1. What decimal part of a pound is 10 oz.? 14 oz.?

2. How many feet are there in $62\frac{2}{3}$ yd.?

3. Write as mixed numbers: 42.5, 1.075, 32.125.

4. Arrange the following in pairs of equal value: .04, $\frac{3}{4}$, .35, $\frac{3}{4}\%$, $\frac{1}{25}$, .005, 75%, .0075, $\frac{1}{2}\%$, $\frac{7}{20}$.

5. Change $\frac{13}{32}$ to a per cent correct to the nearest tenth.

6. Does $\frac{1}{4}$ of 1% of \$300 equal \$7.50, \$.75, or \$75?

7. How many bushels and pecks are there in 158 pecks?

8. Find the exact number of days between Nov. 4 and Feb. 21 of the following year.

Estimate the answers to the following examples; check by working the examples:

9. 41×78 **10.** \$88.73 − \$69.15 **11.** $9\frac{1}{2}\%$ of 2162

12. $16 = \frac{8}{9}$ of ? **13.** $12 = \frac{1}{4}\%$ of ? **14.** $.8 = \frac{?}{25}$

15. $2.4 = \frac{3}{4}$ of ? **16.** $28 = 400\%$ of ? **17.** $.4 = ?\%$ of .12

18. What number increased by 25% of itself equals 95?

19. Multiply 523.01 by 17.5, and divide the result by $7\frac{1}{2}$.

20. 5% of 258 = 3% of what number?

21. Find the average of $121\frac{2}{3}$, $97\frac{5}{6}$, $79\frac{1}{4}$, and $104\frac{3}{8}$.

22. If p = \$376, $r = 3\frac{1}{2}\%$, and t = 7 mo., find i.

23. At 3% a year, how long would it take \$2000 to earn \$50 interest?

24. Find the interest on \$450 for 96 da. at 5%, using three different methods.

Each of the following problems has one or more numbers that you will not need to use in finding the answer. Be ready to tell what numbers are needed. Then work the problems.

1. A train leaves Detroit at 7:29 P.M. and arrives at Buffalo, 251.7 mi. distant, at 12:04 A.M. How long does the run take?

2. Mr. Betts owes Mr. Yates \$3500 on a \$9500 citrus grove which Mr. Yates sold him. Last year, Mr. Betts paid \$192.50 interest on the debt. What rate of interest did he pay?

3. In $4\frac{1}{2}$ da., Mr. Drew plowed $13\frac{1}{2}$ A. of a 24-A. field. At that rate, how long would it take him to plow a field of 36 A.?

4. On Feb. 1, you buy from A. B. Jones merchandise costing \$1950, paying \$1000 cash and giving your note for \$950 at 5%, to be paid at the end of 6 mo. What amount must you pay Mr. Jones at the end of 6 mo.?

5. A poultry dealer in a large city bought 65 turkeys through an agent. The turkeys weighed 570 lb., the price per pound was $22\frac{1}{2}¢$ delivered, and the agent's commission was 5%. What was the total cost of the turkeys to the poultry dealer?

6. Parker Bros. make a profit of 81¢ a dozen on screw drivers which cost them \$3.51 a dozen. In figuring the selling price, they allow 15% for profit and 20% for overhead, based on the selling price. What is the selling price per dozen? How much is this apiece?

Try to work these problems without using a pencil and paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. What is the premium on a $1000 fire insurance policy at the 3-year rate of $1.25 per $100?

2. Mr. Arnold took out an annuity that would pay him an annual income of $1000 at a later age. The annual premium rate was $66.04 per $100 of annual income. How much was Mr. Arnold's premium?

3. Mr. Small took out a $3000 life insurance policy at $32 per $1000. What is his yearly premium?

4. How much can Mr. Morris borrow on his $4000 life insurance policy if the loan value is $116 per $1000?

5. A store valued at $12,000 is insured against fire for $\frac{2}{3}$ of its value. Fire damages the store to the extent of $7250. How much should the insurance company pay?

6. Find the yearly premium on Mr. Caslin's $2000 fire insurance policy at the rate of $\frac{3}{4}\%$.

7. A house worth $8000 is insured against fire for $\frac{3}{4}$ of its value. The insurance rate is 25¢ per $100. What is the yearly premium?

8. Mr. Norton insured his truck for $1200 against fire and theft at 85¢ per $100. How much did the insurance cost him?

9. A man paid 80¢ per $100 for fire and theft insurance on his new $1600 car, plus $47.20 for public liability insurance and $12.10 for property damage insurance. How much did the total insurance cost him?

1. Mrs. Frazer bought a purse for \$4.25, which was 85% of the regular price. What was the regular price?

2. A mattress which usually sells for \$37.50 is sold for \$28.50. Find the per cent of discount allowed.

3. Mr. Bates bought a camp site for \$550. Two years later, he was able to sell the property at a profit of 132%. How much did he receive for the land?

4. Find the exact interest on \$396 at 6% from May 4 to July 13.

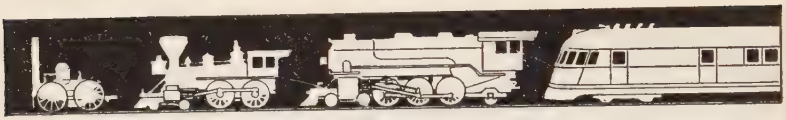
5. An article costing \$32.50 is sold for \$40. Find the rate of profit based on the cost. On the selling price.

6. A 90-day non-interest-bearing note for \$1260, dated Oct. 4, is discounted on Oct. 10 at 6%. Find the discount and the proceeds.

7. Successive discounts of 25% and 5% are allowed from a bill amounting to \$624. Find the net amount of the bill. How much was paid if a cash discount of 2% was taken?

8. An automobile that is bought for \$980 depreciates in value 30% the first year and an additional 10% of the purchase price the second year. What is the value of the car at the end of the first year? The second year?

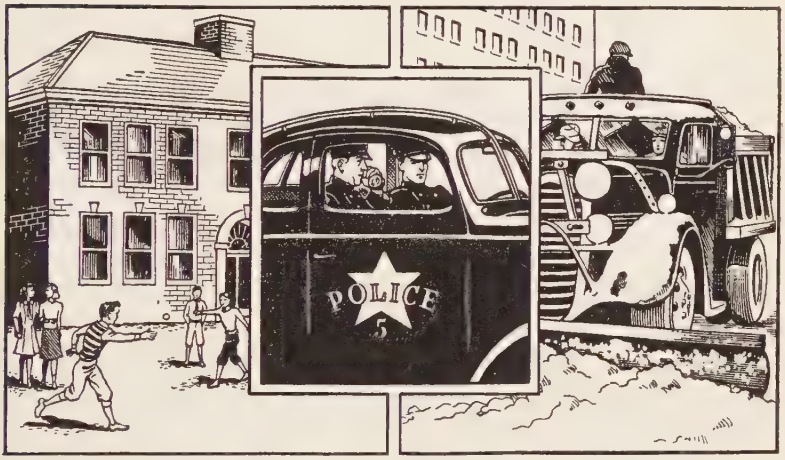
9. A commission merchant received \$450.60 for a load of apples shipped to him by a fruit grower. He paid \$29.72 freight, \$18.85 storage, and \$13.50 for hauling. His commission was 5%. How much should he remit to the fruit grower?



1. Add: 4.6, .035, 7.28, 24, 8.084, 16.0025.
2. Multiply 8.209 by 7.09.
3. $8\frac{1}{2} \times 16\frac{1}{4} \div 2\frac{1}{2} = ?$
4. $18\frac{3}{4} + 5\frac{3}{5} - 2\frac{1}{4} = ?$
5. $83.84 \div 6.4 = ?$
6. Divide 7 gal. 3 qt. 1 pt. by 4.
7. $p = \$12{,}500$; $i = \$312.50$; $t = 1$ yr. Find r.
8. Find the interest on \$688 for 39 da. at 6%.
9. Find the cost of fire and theft insurance for 1 yr. on an \$860 automobile at 85¢ per \$100.
10. Find the yearly premium on a 20-payment life policy for \$6500 if the premium rate is \$34.13 per \$1000.
11. At age 25, a man takes out an ordinary life policy for \$4000, at \$21.49 per \$1000. He dies at 67. How much more does his beneficiary receive than was paid in premiums?
12. A building valued at \$24,000 was insured for $\frac{2}{3}$ of its value, at \$.675 per \$100, for a term of 3 yr. What was the cost of insurance per year?
13. How much can Mr. Daly borrow on his \$4500 endowment policy when the loan value is \$289 per \$1000?
14. How much must be set aside each month to pay the yearly premium on a \$5000 life insurance policy if the rate of premium is \$38.78 per \$1000?
15. Starting at age 60, a retirement annuity policy taken out at age 32 will pay a monthly income of \$22.75 for each \$100 of annual premium paid. Find the monthly income when the annual premium was \$800.

*Record your score on your Progress Chart. See page 321.

UNIT V. TAXES: HOW WE SUPPORT PUBLIC SERVICES



Some of the ways a city serves its people.

All night, the snow fell. But by the time Jack started for school, the city's snowplows had cleared most of it away. The roads, however, were icy; and policemen were busy directing traffic and helping school children safely across the streets.

At school, Jack's teachers instructed him in his lessons and in gymnasium work. With other pupils, he was examined by a medical inspector. During recess, he played in the school playground and watched with excitement as city firemen worked to put out a blaze that had broken out in a store near by.

After school, Jack coasted in one of the city's parks, and later stopped at the public library to get a book.

But never once did Jack ask himself, "Who pays for all these services the city is giving us?"

In the United States, we have three divisions of government—**local, state,** and **federal** or national.

Local governments are those of county, city, and town. Cities and towns provide fire and police protection, paved streets, public schools and libraries, law courts, welfare service, water supply, parks and playgrounds, and waste disposal. Counties maintain county highways, hospitals, law courts, and homes for old needy people.

Our state governments look after the building and repair of state highways, bridges, and waterways. They make and enforce the state laws, and provide state universities and teachers' colleges, state courts, prisons, and other institutions. State police patrol the highways, protect rural communities, and help in emergencies. Besides these things, the state governments safeguard health by regular inspection of hotels, restaurants, water supplies, and packing plants; and give financial aid to counties, cities, and towns.

The federal government provides for a body of lawmakers to make the national laws and also sees that these laws are enforced. It maintains the army and navy for national protection, and operates the postal, or mail, service. It protects the national forests; aids agriculture; gives relief to communities hit by disasters; and gives state aid in times of depression. It provides pensions for soldiers and sailors, and also provides for old-age insurance.

Vast sums of money are needed to carry on the work of the various governments, and this money is raised by **local, state,** and **federal taxes.**

Before a local, state, or national government can tell how much must be raised by taxes, an **annual budget,** or list of probable expenses, must be made. This budget is based on the past year's expenses and on estimates of what the various departments will need during the coming year.

Much of the money needed by local governments is raised by a tax on **real estate** or on real estate and **personal property.** A man's real estate is the land and buildings he owns. His personal property is his movable property, such as furniture, automobiles, and livestock.

When a city or town has made out its budget and knows how much money it needs, it must determine what share of the money each property owner should pay.

City tax officers, called **assessors,** estimate the value of each piece of property to be taxed. This **assessed valuation** may be a fair market value for the property or a per cent of its market value, as 50% or 80%. The property is then taxed, the tax rate depending upon the total assessed value of all the property in the city or town and the amount of money needed as shown by the budget.

How to Find the Tax Rate

The total assessed valuation of the taxable property in the town of Winthrop is $2,450,000. The amount to be raised by taxation is $66,150. Find the tax rate.

$$2450000\overline{)66150.000} \quad .027 = 2.7\% \text{ tax rate}$$

To find the tax rate, divide the amount of taxes to be raised by the total assessed valuation.

Remember: $\text{Tax rate} = \dfrac{\text{Total taxes to be raised}}{\text{Total assessed valuation}}$

Find the tax rate in each of the following:

	To Be Raised	Valuation		To Be Raised	Valuation
1.	$1680	$56,000	4.	$66,420	$1,620,000
2.	$6240	$240,000	5.	$186,124	$5,890,000
3.	$18,850	$580,000	6.	$182,700	$6,525,000

The tax rate may be expressed in four ways:

(1) As a per cent of the assessed value, as 2.7% of $2,450,000.

(2) (3) As dollars and cents per $100 or $1000 of assessed value, as $2.70 per $100 or $27.00 per $1000.

(4) As *mills* per dollar of assessed value, as 27 mills per $1. A mill is $\frac{1}{10}$ of a cent, or $.001. 27 mills = $.027.

1. Express these rates in three other ways: (a) $3.50 per $100; (b) $2\frac{1}{2}\%$; (c) 32 mills per $1; (d) $28.75 per $1000.

How to Find the Tax on Property

Mr. Morgan owns property assessed at $7500. Find the tax he must pay if the tax rate is expressed as: (1) $2\frac{1}{2}\%$; (2) $2.50 per $100; (3) $25 per $1000; (4) 25 mills per $1.

1. Tax Rate Expressed as a Per Cent

$7500 × .025 = $187.50 tax

Multiply the assessed valuation by the per cent of tax expressed as a decimal.

2. Tax Rate Expressed as Dollars and Cents per $100

$7500 ÷ $100 = 75
$2.50 × 75 = $187.50 tax

Multiply the rate per $100 by the number of hundreds of dollars in the assessed valuation.

3. Tax Rate Expressed as Dollars and Cents per $1000

$7500 ÷ $1000 = 7.5
$25 × 7.5 = $187.50 tax

Multiply the rate per $1000 by the number of thousands of dollars in the assessed valuation.

4. Tax Rate Expressed as Mills per Dollar

25 mills = $.025
$.025 × 7500 = $187.50 tax

Express the mill rate as a decimal part of $1. Multiply this rate by the number of dollars in the assessed valuation.

Find the taxes at the rates given. In figuring taxes, count any fraction of a cent in your answer as 1¢.

	Valuation	Per Cent	Per $100	Per $1000	Mills per $1
1.	$6000	$2\frac{3}{4}\%$	$2.65	$32.60	38
2.	$16,000	$1\frac{3}{4}\%$	$3.80	$22.90	27
3.	$8500	$3\frac{1}{5}\%$	$4.25	$36.50	24

PENALTY FOR LATE PAYMENT OF TAXES

If a property owner fails to pay his taxes when due, he may have to pay an increased amount as a **penalty.** In some cities, the penalty is a certain per cent of the amount of tax due. In others, interest is charged on the amount of tax for the overdue time.

1. When Mr. Connor paid his overdue tax bill of $128.50, he paid a penalty of 5%. How much did he pay in all?

2. Mr. Kent's tax bill of $64.80 was due on Dec. 1. He did not pay until Mar. 1. If he was charged 1% a month for the overdue time, how much did he pay in all?

SPECIAL ASSESSMENTS

A city often makes **special assessments** against property owners for improvements that increase the value of their property. The cost of a new sewer or a new pavement may be charged in whole or in part to those property owners who benefit from the improvement. The special assessment must be paid in addition to the regular taxes, but payment may be spread over a number of years.

1. A city assessed property owners in a new section $3 a front foot for the cost of laying a sewer. The city paid half the cost of new pavements, and the property owners were assessed the other half, which amounted to $3.85 a front foot. Mr. Newcomb's property in the section is 60 ft. across the front. How much did he pay for sewer and paving?

1. In many rural districts, the school tax is a separate tax. The school tax in one district is 6 mills per $1. How much is the school tax on property assessed at $8246?

2. Mr. Richards was assessed $12.75 per front foot for the grading and paving of the street in front of his house. If his lot is 65 ft. wide, how much was his assessment?

3. Mr. Dean owns real estate assessed at $6500 and personal property assessed at $1275. How much will his total property tax be if the rate is $32.76 per $1000?

4. A town needed to raise $51,000 for school expenses. The total assessed valuation of property was $3,000,000. Find the school tax on property assessed at $4800.

5. My property cost me $8000. It is assessed at 65% of its cost. My taxes this year amount to $130. What is the rate per $1000 of assessed valuation?

6. A man owns a house worth $6000, which is assessed at 75% of its value. If the tax rate is $26.74 per $1000 of assessed valuation, how much will his tax be?

7. Mr. Newton paid a tax of $4.42 per $100 on property assessed at $12,500, plus a penalty of 8% of the amount of his tax for late payment. How much did he pay?

8. A school district with an assessed valuation of $2,886,400 required $36,080 for its schools. What was its school tax per $1000? Per $100? Find the school tax on a $6000 property assessed at 60% of its value.

9. In Carr City, property is assessed at 55% of its actual value, and the tax rate is $5.13 per $100. In Benton, property is assessed at 80% of its actual value, and the tax rate is $3.84 per $100. In which of the two cities will the tax be less on property that has an actual value of $4000? How much less?

1. Why should all the people in a community share in the cost of its government?

2. Churches, public buildings, schools, and charitable institutions are not taxed. In each case, tell why these properties are not taxed.

3. Does the man who rents a house help pay the taxes on it? Explain your answer.

4. If the amount of taxable property in a city increases, what effect does that have on the tax rate, provided the budget expenses do not change?

5. Discuss wise economies which a city government might practice so as to reduce the tax rates. Would leaving the streets unrepaired be a wise economy?

6. What other sources of income does your community have besides a tax on property? Discuss each one, and tell whether the income it produces is large or small.

7. What is the tax rate in your community this year? Is it higher or lower than last year's rate? Give the reason for the difference. How does the rate in your community compare with the rate in some neighboring community? See if you can give the reasons for any difference.

8. Are the people in your community allowed a discount for paying tax bills before they are due? Find out what the discount and the conditions of payment are.

9. Which three departments of your local government are given the largest sums from the tax money. Why?

10. If possible, bring several kinds of tax bills to class, and discuss them.

Add:

1.	2.	3.	4.	5.	6.
$7\frac{4}{5}$	$23\frac{3}{8}$	$5\frac{5}{6}$	$12\frac{3}{8}$	$18\frac{4}{5}$	$10\frac{7}{8}$
$5\frac{9}{10}$	$9\frac{9}{16}$	$11\frac{3}{5}$	$4\frac{2}{3}$	$16\frac{1}{2}$	$8\frac{5}{12}$
$14\frac{1}{2}$	$6\frac{3}{4}$	$12\frac{2}{3}$	$27\frac{1}{2}$	$2\frac{5}{6}$	$22\frac{5}{6}$

Subtract:

7.	8.	9.	10.	11.	12.
16	$4\frac{1}{8}$	$16\frac{2}{5}$	$21\frac{7}{10}$	$39\frac{1}{4}$	$45\frac{9}{16}$
$4\frac{7}{12}$	$\frac{7}{24}$	$14\frac{1}{2}$	$8\frac{5}{6}$	$28\frac{3}{5}$	$17\frac{2}{3}$

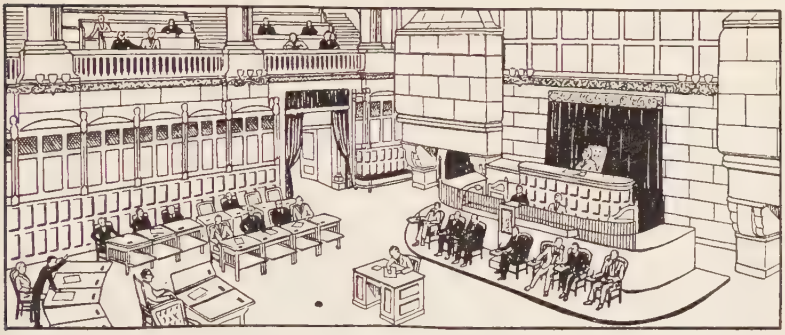
Find the product of:

	A	B	C	D
13.	$\frac{3}{4} \times \frac{5}{9}$	$6\frac{1}{8} \times 36$	$10\frac{1}{2} \times 5\frac{1}{3}$	$5\frac{5}{6} \times 4\frac{1}{5}$
14.	$11\frac{1}{3} \times 3\frac{1}{2}$	$\frac{5}{12} \times \$256$	$\frac{2}{3} \times 12\frac{3}{4}$ bu.	$12\frac{1}{2} \times 12\frac{1}{2}$

Find the quotient of:

15.	$21 \div \frac{3}{4}$	$\frac{15}{16} \div 6$	$\frac{5}{6} \div \frac{3}{5}$	$10\frac{2}{3} \div 2\frac{1}{4}$
16.	$3\frac{1}{3} \div 6\frac{1}{9}$	$8\frac{3}{5} \div 4\frac{3}{10}$	$26\frac{5}{8} \div \frac{1}{2}$	$4.8 \div 3\frac{3}{4}$

17. From the sum of $2\frac{1}{4}$ and $3\frac{3}{8}$, subtract $\frac{5}{16}$.
18. Subtract the sum of 5 and $8\frac{3}{5}$ from $16\frac{1}{4}$.
19. Multiply the sum of $6\frac{1}{4}$ and $1\frac{1}{3}$ by $4\frac{1}{2}$.
20. Subtract $13\frac{3}{4}$ from 16; multiply the difference by $9\frac{1}{3}$.
21. Multiply the quotient of $4\frac{3}{8} \div 2$ by $\frac{2}{3}$.
22. Divide the sum of $7\frac{1}{3}$ and $3\frac{5}{6}$ by 4.
23. Divide the sum of $4\frac{2}{3}$ and $2\frac{2}{5}$ by $\frac{5}{6}$.



State lawmakers at work.

Did you ever go to the capitol of your state, where the state laws are made and the state's business is directed? Did you wonder where all the money comes from to pay the state representatives, the judges of the state courts, and the great number of people at work in the different departments? When you think of all this expense added to the cost of the many other services each state gives its people, it is not surprising that a state must raise large sums of money from many sources.

Each state has its own ways of raising funds, but perhaps the most common means are the following:

1. **Income Tax.** A tax on personal incomes.
2. **Franchise Tax.** A tax on corporations for the privilege of doing business.
3. **Corporation Taxes.** Taxes on the amount of business done by corporations.
4. **Property Tax.** A tax on real estate.
5. **Sales Tax.** A tax on retail sales.
6. **Gasoline Tax.** All states tax purchases of gasoline.
7. **Inheritance Tax.** A tax on property received through the death of another.
8. **Licenses.** Fees for car licenses; drivers' licenses; licenses to hunt, fish, etc.

This page gives in round numbers the sums which one state collected from its various sources of income in a recent year, and the ways the state spent this income. The graphs show what part of each income dollar was received from each source, and what part of each dollar was spent for each expenditure.

STATE INCOME

Income tax	$111,200,000
Corporation tax	78,700,000
Gasoline tax	59,300,000
Driving and car licenses	37,300,000
Inheritance tax	34,000,000
Other taxes	68,600,000

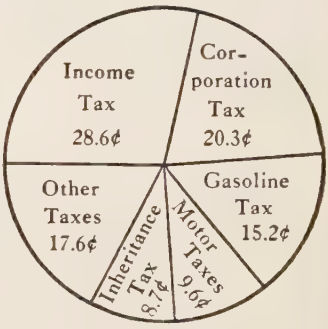
STATE EXPENDITURES

Aid Granted Communities

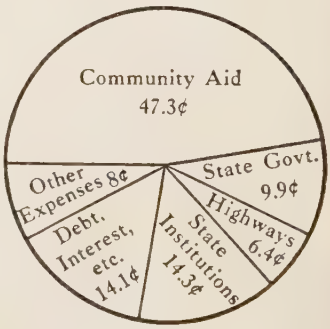
Education	$122,600,000
Health	700,000
Public works	5,400,000
Social welfare	47,200,000

Regular State Expenses

State government	$36,800,000
Highways	24,000,000
State institutions	53,200,000
Debt, interest, etc.	52,500,000
Other expenses	29,900,000



State Income Dollar



State Expenditure Dollar

1. Find the state's total income and its total expenses.
2. Show by a bar graph the regular state expenses.
3. Read the graphs, substituting per cents for cents.
4. If your state places a tax on personal incomes, bring to class a state income tax form and make a class study of it.

Some states and some communities place a tax on retail sales. The merchants collect these taxes from the purchasers and turn the money over to the state or community.

The articles that are taxed and the rate of taxation vary in different places. Most states place no tax on food except that served in restaurants.

A common **retail sales tax** rate is 2% of the sale. At this rate, if you buy a hat for \$3.00, a tax of 2%, or 6¢, is added, and you must pay \$3.06.

Find the amount, including a 2% sales tax, paid for each of the following:

1. Gloves priced at \$1.50
2. Overcoat priced at \$16.00
3. Dress priced at \$14.50
4. Piano priced at \$960
5. Gas range priced at \$42
6. Book priced at \$2.25

One city has a sales tax expressed as follows:

Sales under 12¢, free from tax.

Sales of 12¢ to 63¢, 1¢ tax.

Sales of 63¢ to \$1, 2¢ tax.

Sales of \$1 and over, 2% tax on even dollars plus the above rates on any part of a dollar. On a sale of \$3.64, the tax would be 6¢ + 2¢, or 8¢.

Find the sales tax at the above rates on purchases of:

7. \$.68	**9.** \$2.00	**11.** \$.95	**13.** \$2.45	**15.** \$1250
8. \$.10	**10.** \$5.40	**12.** \$3.05	**14.** \$6.28	**16.** \$164.16

17. Jeanne bought 4 yd. of linen @ 56¢. Including a sales tax of 2%, how much did she pay?

18. Bill buys a suit priced at \$14.75, and a shirt marked \$2.98. How much does he pay for each if there is a sales tax of 3%?

Gas 11¢—Tax 5¢ reads this filling station sign. This shows one form of sales tax actually at work. Every gallon poured into a car at this station costs the driver 16¢. Of this amount, 11¢ pay for the gas, 4¢ go to the state in which the gas is sold, and 1¢ goes to the federal government.

The tax on gasoline and the uses made of the money collected vary in the different states. Some states use all the state gas tax money in building and repairing state highways. Some use part of it for other purposes. Every state collects the 1¢ per gallon for the federal government.

1. Mr. Barlow's car averages 14 miles to a gallon of gas. The total gas tax in his state is 4¢ a gallon. Figure his gas tax for 1253 miles of driving.

Following are four examples. In each case, find: (a) the cost to each driver of the gas purchased; (b) the total tax on each purchase; (c) how far each driver could go at $12\frac{1}{2}$ miles to a gallon; (d) the cost of gasoline per mile.

	Gal. bought	Price per gal. without tax	State tax per gal.	Federal tax per gal.
2.	12	13¢	4¢	1¢
3.	20	12¢	7¢	1¢
4.	15	17¢	5¢	1¢
5.	18	15¢	3¢	1¢

6. In a recent year, the people of our country paid 757 million dollars in gasoline taxes. The number of gallons of gasoline sold was 21,346 millions. Find to three decimal places the average tax per gallon.

1. A filling station sold in one day 450 gal. of gasoline. If the state gasoline tax was 4¢ per gallon, how much did the state tax amount to on the day's sales?

2. What will be the cost of license plates for an automobile which weighs 3300 lb. if the cost of the plates is 50¢ for each 100 lb.?

3. A taxi company paid \$5 each for 9 new three-year operator's licenses, and \$4 each for 26 renewals. What was the total amount paid by the company for the licenses?

4. For the privilege of showing motion picture films in a certain state, a film company paid \$3 a thousand feet for original film and \$2 a thousand feet for copies. How much was the total tax on 15,000 ft. of original film and 85,000 ft. of copy?

5. In one community, 16 real-estate dealers paid \$25 each for state licenses allowing them to do business; and 62 of their salesmen took out similar licenses at the rate of \$2 each. What was the total amount received from the sale of these licenses? One half of this total went to the local community and one half to the state. How much did each receive?

6. To raise money for advertising to increase the sale of milk, one state placed a tax of $\frac{3}{4}$¢ on each 100 lb. of milk sold by dairy farmers. What tax did a dairy farmer pay on sales of 20,000 lb. of milk?

7. When Mr. Carson died, he left an estate valued at \$185,000. Of this amount, \$20,000 was **exempt,** or free from tax, according to the state law. An inheritance tax had to be paid on the remainder at the following rates: 1% on the first \$25,000; 2% on the next \$25,000; and 4% on the balance. Find the amount of the inheritance tax.

Five people went on an automobile trip that lasted two days. Using the following information about the trip, answer the questions at the bottom of the page. The accuracy of your work will depend largely on your choice of the right figures to be used in finding each answer.

Distance traveled, 600 mi.

The car averaged 15 mi. per gallon of gasoline. The average price of gasoline was 18¢ a gallon.

The car averaged 300 mi. per quart of oil. Oil cost 25¢ a quart.

When new, the tires cost $60 for a set of 4. The type of tire used has an average life of 18,000 mi.

Meals cost as follows per person a day: breakfast, 50¢; lunch, 50¢; dinner, 75¢.

Rooms cost $1.50 per person a night.

Amusements cost $15; car repairs, $5.

1. How many gallons of gasoline were used?
2. Find the cost of the gasoline used.
3. How many quarts of oil were used?
4. Find the cost of the oil used.
5. What was the cost of wear on the tires?
6. Find the total cost of meals for the 2 days.
7. Find the total cost of rooms for the 2 nights.
8. What was the total cost of the trip?
9. What was the average cost of the trip per person?
10. What was the cost of the trip per mile?

Where our federal tax laws are made.

The federal government requires enormous amounts of money to maintain the army and the navy, enforce the laws passed by Congress, and provide the many other services needed by the country which are not provided by the local and state governments.

1. In a recent year, the cost of running our federal government was $7,700,000,000. Of this amount, $2,260,000,000 was spent for relief and business recovery, $1,556,000,000 for national defense and care of veterans, $460,000,000 for public works, and $570,000,000 for running the different departments of the government. Show these facts in a bar graph.

The most important sources of federal income are **internal revenue,** which is income from sources within the country; and **customs,** or **import duties,** which are taxes on goods brought into the United States from foreign countries.

Internal revenue includes taxes on the incomes of persons and corporations; taxes on the manufacture and sale of tobacco, automobiles, and other products; inheritance taxes; and taxes on entertainment tickets, long-distance telephone messages, and telegraph and cable messages.

2. In a recent year, the income of the federal government amounted to $6,240,000,000. Of this amount, $359,000,000 came from customs and $2,635,000,000 from income taxes. What per cent of the income came from each source?

By March 15 of each year, a **federal income tax return,** or report, for the preceding year must be made and filed by every unmarried citizen or resident of the United States who received a **net income** of $1000 or more during that year and by every married couple who received $2500 or over. A net income is the amount left after business expenses and other deductions which are allowed have been subtracted from the total income.

When Mr. Jordan made out his federal income tax return last year, he first listed his income from salary, interest, etc., to find his total income.

Next, he listed the amounts which he was allowed to deduct according to the instructions on the tax form. His total income minus the deductions gave him his net income.

Mr. Jordan was married and had two small children. According to the tax form, a married person is entitled to an exemption of $2500 for himself and his wife, plus a credit of $400 for each child and each dependent other than his wife under 18 years of age. A single person has a personal exemption of $1000.

Income

Salary	$3600
Interest on loans	600
Interest on investments	200
Total income	$4400

Deductions

Gifts to charity	$ 100
State and local taxes	225
Interest on a debt	72
	$ 397

$4400 − $397 = $4003, net income

Exemptions and Credits

Married	$2500
2 children	800
Earned income credit (10% of $3600)	360
	$3660

Besides deductions and exemptions, Mr. Jordan found that the government allowed him a credit of 10% of his **earned income** — money received for personal services.

Mr. Jordan's net income less exemptions and credits gave his **taxable balance,** on which a tax of 4% had to be paid.

The federal income tax rate of 4% of a taxable balance is the *normal* federal tax rate. An additional tax, called a *surtax,* is charged on net incomes over a certain size. The surtax rate increases with the size of the net income. Large incomes are thus taxed at a much higher rate than small incomes.

How to Find the Normal Federal Income Tax

Mr. Jordan's total income was \$4400. His deductions were \$397, and his exemptions and credits were \$3660. What was his federal income tax at 4%?

\$4400 total income	From the total income, subtract the deductions to find the net income.
— 397 deductions	
\$4003 net income	From the net income, subtract the exemptions and credits to find the taxable balance.
—3660 exemptions and credits	
\$ 343 taxable balance	
\$343 × .04 = \$13.72 normal federal income tax	Multiply the taxable balance by 4% to find the normal federal income tax.

Find the normal federal income tax on each of the following incomes, allowing an earned income credit of 10%:

	Earned Income	Other Income	Deductions	Exemptions
1.	\$4000	\$350	\$320	\$1000
2.	\$4250	\$300	\$275	\$2500
3.	\$3640	\$1700	\$1220	\$2900

4. Dr. Conway earned \$5200 last year. His income from investments was \$1665. In figuring his income tax, he deducted \$90 for interest paid on a debt, \$275 for taxes paid, \$1200 for business expenses, an exemption of \$1400 as an unmarried man with one dependent, and a credit of 10% of earned income. Find his income tax.

The list of customs, or import duties, which are charged on goods brought into our country is known as the **tariff.** The law which fixes the amounts of duty to be charged is called a **tariff law.** The purpose of a tariff law is not only to raise revenue, but also to protect industries in our country from the competition of goods made more cheaply abroad.

There are two kinds of import duties: **ad valorem** and **specific.** An ad valorem duty is a per cent of the invoiced value of the article imported, as 45% or 70%. A specific duty is a charge per unit of measure, as 9¢ a pound, $6 a dozen, 42¢ a bushel, etc. On some goods, both an ad valorem and a specific duty are charged. Many goods are exempt from duty and are on the **free list.** In recent years, each tourist has been allowed to bring into the country, free of duty, $100 worth of goods for his own use.

Duties are figured on the nearest whole dollar. Find the duty on each of the following imports:

Ad Valorem Duties

1. Lace valued at $460.47 — Duty, 90%
2. Toys valued at $385.60 — Duty, 45%
3. Imitation pearls valued at $1290 — Duty, 60%

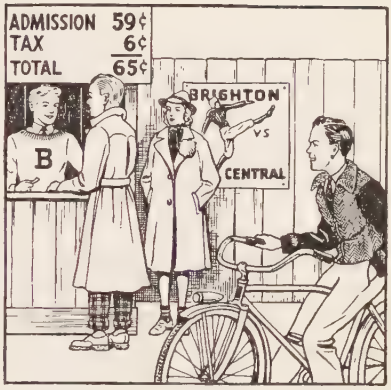
Specific Duties

		Duty			Duty
4.	96 doz. eggs	5¢ doz.	6.	2000 lb. nuts	5¢ lb.
5.	1800 lb. lemons	$2\frac{1}{2}$¢ lb.	7.	1600 bu. wheat	42¢ bu.

8. Find the duty on $2\frac{1}{2}$ T. of book paper valued at $7.70 per cwt., at 5% ad valorem and $\frac{1}{5}$¢ per pound specific.

9. Mr. Martin paid a duty of 40% on 215 sq. yd. of carpet invoiced at $752.50. For how much per square yard would he have to sell the carpet to include the cost, the duty, 25% for transportation expense and overhead, and 10% for profit, based on the selling price?

At the ticket window at a Brighton College football game, Jack noticed that the tickets were selling for 59¢ plus a tax of 6¢ each.



There is a federal tax of 10% on amusement tickets priced at more than 40¢. A fraction of a cent in the tax counts as a whole cent.

1. Find the federal tax and the cost of a ticket when the price without tax is 50¢; 86¢; 77¢; \$1.50; \$2.04.

2. What are the prices, without tax, of tickets that cost as follows including a 10% federal tax: \$3.30? \$3.85?

3. At a football game, 2450 people paid \$1.10 each for admission tickets, 3600 paid \$2.20 each, and 2860 paid \$2.75 each. If these prices included a 10% tax, how much was the total tax paid?

4. Mr. Logan sold his home for \$7750. On the deed which transferred the property to the new owner, he was required to pay a stamp tax of 50¢ for each \$500 or fraction thereof of the sale price. How much stamp tax did he pay?

5. An automobile sells for \$864 at the factory. This includes a federal tax of 3%. How much is the tax?

6. When Mr. Fenwick died, he left an estate valued at \$37,500 after all exemptions and other allowances had been deducted. The federal inheritance tax was 2% of the first \$10,000, 4% of the second \$10,000, 6% of the third \$10,000, and 8% of the balance. Find the total tax.

Many of the taxes charged against manufacturers and producers are added by them to the selling prices of their goods and are thus passed on to the consumer, or buyer. Such taxes are called **hidden,** or **indirect, taxes** because the consumer pays them indirectly, sometimes without realizing that he is paying any tax at all.

Although people who own little or no property and whose incomes are small may pay few direct taxes, a considerable part of their incomes pays hidden taxes. It has been estimated that a person whose weekly income is only $20 pays as much as $2.40 a week in hidden taxes, while a person with a weekly income of $50 may pay as much as $6.20 a week.

1. According to the facts given in the above paragraph, what per cent of an income of $20 a week goes for hidden taxes? Of an income of $50 a week? How do these per cents compare with the property tax rate in your community? With the normal federal income tax rate?

The largest number of hidden taxes are paid on necessities and everyday commodities. As much as 19¢ of every dollar spent by the housewife on groceries and meats pays for hidden taxes. The price of a loaf of bread includes 53 hidden taxes which have been passed on to the consumer by the farmer, the grain elevator, the flour mill, the railroad, the flour trucker, the bread wrapper manufacturer, and the bakery which produced the bread.

2. If the hidden taxes on bread amount to an average of 2¢ a loaf, what per cent of the price of bread in your community do such taxes represent?

3. Out of a monthly rent of $38 paid by a family, $9.92 pays hidden taxes. What per cent are these taxes of the total rent? Of $22 spent for clothing, $2.09 pays hidden taxes. What per cent are these taxes of the total spent?

By land, sea, and air, the mail speeds on its way.

The postal service furnished by the federal government is an enormous business which handles millions of letters and packages each day. The postal business is not organized for profit, but to give the greatest service for the least money. A fee of 3¢ will carry a letter from New York to Alaska, from Duluth to Chile, from San Francisco to Argentina. How far would you be willing to carry a letter for 3¢?

1. Although the Post Office Department receives large sums of money from the sale of stamps, the expenses of the department are usually more than the income. The amount short, called a **deficit,** is paid from federal tax receipts. In a recent year, the postal service cost $772,743,145 and its income was $726,201,110. What was the deficit?

2. In the past ten years, the postal route mileage of the United States increased from 478,000 mi. to 520,000 mi. Find to the nearest tenth the per cent of increase.

3. Find out the cost of sending: (a) a letter weighing $1\frac{1}{2}$ oz. to England; (b) an air-mail letter weighing 1 oz. to Hawaii; (c) a postcard to China; (d) a $2\frac{1}{2}$-lb. package by first-class, special delivery, to some American city.

Work the examples in column A. If you fail on any example, do the example opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review Pages
1. Finding the Tax Rate (Express It in Four Ways)		
Assessed Valuation: \$85,000; Tax: \$1020 Assessed Valuation: \$1,240,000; Tax: \$46,500	Assessed Valuation: \$64,000; Tax: \$960 Assessed Valuation: \$2,860,000; Tax: \$64,350	439-440
2. Finding the Tax on Property		
Assessed Valuation: \$18,000; Tax Rate: $2\frac{1}{4}\%$ Assessed Valuation: \$12,800; Tax Rate: \$36 per \$1000 Assessed Valuation: \$6400; Tax Rate: \$2.75 per \$100 Assessed Valuation: \$8500; Tax Rate: 16 mills per \$1	Assessed Valuation: \$20,000; Tax Rate: $3\frac{1}{5}\%$ Assessed Valuation: \$10,600; Tax Rate: \$32 per \$1000 Assessed Valuation: \$7500; Tax Rate: \$1.80 per \$100 Assessed Valuation: \$9600; Tax Rate: 15 mills per \$1	440-441
3. Finding the Normal Federal Income Tax		
Earned income, \$4000; other income, \$1260; deductions, \$185; exemptions, \$2900; earned income credit, 10%. Normal tax rate, 4%.	Earned income, \$3000; other income, \$1500; deductions, \$124; exemptions, \$3200; earned income credit, 10%. Normal tax rate, 4%.	452-453
4. Finding Import Duties		
400 clocks valued at \$.95 each. Duties: specific, $27\frac{1}{2}¢$ each, and ad valorem $32\frac{1}{2}\%$.	70 doz. eyeglasses valued at 80¢ a dozen. Duties: specific, 60¢ a dozen, and ad valorem 20%.	454

Give a brief definition of each of these terms:

1. Taxes
2. Customs
3. Tariff
4. Deficit
5. Imports
6. License
7. Real estate
8. Tax rate
9. Exemptions
10. Tax return
11. Assessors
12. Net income
13. Personal property
14. State budget
15. Earned income
16. Internal revenue
17. Franchise tax
18. Assessed valuation

Complete the following:

19. Taxes passed on by the manufacturer to the consumer as part of the selling price are called ___ ___.

20. A tax per unit of measure on imported goods is a ___ duty.

21. Three common sources of state income are ___, ___, and ___.

22. Goods which may be imported without payment of duty are said to be on the ___ ___.

23. A tax on earnings and other income is called an ___ tax.

24. The most important sources of federal income are ___ and ___.

25. A tax charged against a property for some public improvement that increases the value of the property is known as a ___ ___.

26. A tax on the value of imported goods is called an ___ ___ duty.

27. A tax collected from the purchaser when he buys an article at a retail store is called a ___ ___ tax.

Try to work these problems without using pencil or paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. If a tax rate is \$2.84 per \$100, what is the tax rate per \$1000?

2. If a tax rate is \$36 per \$1000, what is the tax rate in mills per dollar?

3. At a rate of 1%, how much would a village tax collector's commission be on taxes amounting to \$4682?

4. At a tax rate of \$15 per \$1000, what is the amount of the tax on property assessed at \$3000?

5. Property worth \$3600 is assessed at $\frac{3}{4}$ of its value. What is the assessed value of the property?

6. If \$5000 is to be raised by taxing property assessed at \$200,000, give the tax rate as a per cent.

7. The Elmwood Dairy bought 450 gal. of gasoline for its trucks. If the state tax was 3¢ per gallon and the federal tax was 1¢ per gallon, what amount of tax was paid?

8. Find the sales tax at 2% on a \$3.50 sale.

9. Mr. Jackson's net income last year was \$3700. His exemptions and credits were \$1700. Give his federal income tax at 4%.

10. Find the duty on \$600 worth of toys imported from Switzerland, if the rate of duty is 45%.

11. Mr. Saxton's taxable income last year was \$2000. The state income tax was 2% of the first \$1000 and 3% of the second \$1000. Find his total state income tax.

Give the missing terms:

1. In depositing money at a bank, a person fills out a ___.

2. An order on a bank to pay a specified amount of money is called a ___.

3. Write in figures: two hundred six and twenty-six hundredths. Then round the number to the nearest tenth.

4. Write one half of one per cent as a decimal and as a per cent.

5. John Blake owes James Morrow $256. Write his 90-day note for the amount, bearing 6% interest. Endorse the note from James Morrow to Thomas Barclay.

6. At 3 for 5¢, how much will a dozen apples cost?

7. What is the ratio of 26 ft. to 10 yd.?

8. What fractional part of $900 is $540? What decimal part? What per cent?

9. Rearrange in order of size, beginning with the smallest: .27, $\frac{3}{8}$, 25%, $\frac{1}{4}$%, .027, .38, 2.5%, $\frac{31}{100}$.

10. Find the exact interest on $890 at 5% for 108 da.

11. Find the cost of an annuity for $1500 annual income if the single premium rate is $2003.40 per $100 of income.

12. What is the interest on a bank deposit of $2517.84 at $2\frac{1}{2}$% compounded semiannually for $1\frac{1}{2}$ yr.?

13. The cost of a ring is $7.25. At what price must it be sold to allow 25% for overhead and $16\frac{2}{3}$% for profit, based on the selling price? What would the profit be?

Make and work a problem calling for:

1. The premium on a fire insurance policy when the face and rate per $100 are given.

2. The per cent of discount when the regular price and net price are given.

3. The net price when successive discounts are allowed.

4. The regular price of an article when the per cent of discount and the net price are given.

5. The selling price when the cost and the per cents of overhead and profit based on the cost are given.

6. The yearly rate of interest when the principal, the interest, and the time are given.

Tell what facts you would need to use in making a problem calling for each of the following. Then make and work the problems.

7. The interest on a loan.

8. The per cent of decrease.

9. The proceeds of a non-interest-bearing note.

10. The commission and net proceeds of a sale.

11. The per cent of profit based on the cost.

12. The per cent of profit based on the selling price.

13. The total cost when buying on commission.

14. The bank discount on an interest-bearing note.

1. The sale price of a hunting coat is \$6.65, which is $87\frac{1}{2}\%$ of the regular price. What is the regular price?

2. Show by a line graph the following annual fire losses in our country for eight recent years: (1) \$500,000,000; (2) \$450,000,000; (3) \$400,000,000; (4) \$270,000,000; (5) \$270,000,000; (6) \$235,000,000; (7) \$265,000,000; (8) \$254,000,000.

3. Find the amount which may be borrowed on a 20-yr. endowment policy for \$2250 after 7 yr. if the loan value at that time is \$248 per \$1000 of the face of the policy.

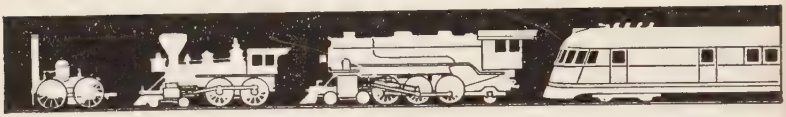
4. Mrs. White paid an annual premium rate of \$44 per \$1000 on a 20-yr. endowment insurance policy for \$5000. (a) What annual premium did she pay? (b) If the total dividends during the term of the policy were \$625, how much in all did the insurance cost her?

5. On Dec. 1, Mr. Peterson had a balance of \$457.60 in his bank. During the month, he made deposits of \$75.48, \$36.92, \$145.60, \$25.35, and \$89.13. He drew checks as follows: \$15.30, \$79.80, \$42.21, \$84.35, and \$76.09. What was his balance in the bank at the end of December?

6. A school teacher receives an annual salary of \$1300. She spends \$650 a year for board, room, and laundry; \$245 for clothing; and \$275 for education and travel. What per cent of her salary is left?

7. A house was sold for \$7820 at a loss of 8% of the cost. For what price should it have been sold to gain 15%?

8. Find the proceeds of a 90-day note for \$1240, without interest, dated June 14 and discounted June 29 at 6%.



1. Add: $17.6 + 126.3 + 701.5 + 6.035 + 56$.

2. From the sum of 5.8 and 14.09, subtract 3.0089.

3. $5.0206 \times 74.35 = ?$

4. $6.273 \div 5.1 = ?$

5. Multiply 5 bu. 3 pk. by 9, and divide the result by 6.

6. 15% of 240 equals 8% of what number?

7. 250% of 18 equals what per cent of 200?

8. Find the interest on \$532 for 48 da. at 5%.

9. At age 48, \$1000 will buy an annual annuity income of \$48.18. At this rate, how much will an annual income of \$1200 cost?

10. A house worth \$6300 is assessed for $\frac{2}{3}$ of its value. Find the property tax to be paid if the rate is \$25 per \$1000.

11. A rural school district with an assessed valuation of \$45,000 needs to raise \$750. What will be the school tax expressed as a per cent?

12. How much will a 2% sales tax produce on a year's sales by a department store which amount to \$1,517,450?

13. A trucking company used 28,460 gal. of gasoline last year. If the state tax was 4¢ a gallon and the federal tax 1¢, what total gasoline tax did the company pay?

14. Last year, Mr. Gordon's earned income was \$5680. Allowing a \$2500 exemption and a 10% credit of earned income, find his federal income tax at 4%.

15. Find the duty on 4200 lb. of book paper, valued at \$280, at 5% ad valorem and $\frac{1}{5}$¢ per pound specific.

*Record your score on your Progress Chart. See page 321.

UNIT VI. PUTTING MONEY TO WORK—INVESTMENTS

Money put to work gathers more money.

To **invest** money means to put it to work in such a way hat it brings you more money or other returns of real value. When you put money in a savings account, it earns com-ound interest. Money spent for business training or a col-ege education should prove a good investment, as a person vith a better education usually can command a higher salary han one who hasn't had that training.

An automobile may be a good investment for a salesman vho has to travel a great deal. Why? In what ways may a ome be a good investment? How could money spent in ravel bring a return? Is insurance a good investment? Why?

All of us do not have money to invest, but we have time nd energy which we can use wisely. Write on the black-oard a list of ways in which a girl or a boy can use time nd energy so that they become good investments. Dis-uss the returns which each investment would bring.

James Kennedy wanted to be an electrical engineer. When he left high school he took a special college course which would fit him for the work. After college, James worked for several years with a firm of engineers until he had the experience he felt necessary for opening an office of his own. Then, with his savings, James started in as the **owner** of a small business.

Later, James and a friend, John Hart, who was also an electrical engineer, decided to go into business together. So they formed a **partnership** under the name KENNEDY & HART, the expenses and the profits (or losses) to be shared equally.

In their spare time, James and John worked to perfect a new type of electrical motor that James had designed. At last the new motor was finished, and after repeated tests it was found to be successful.

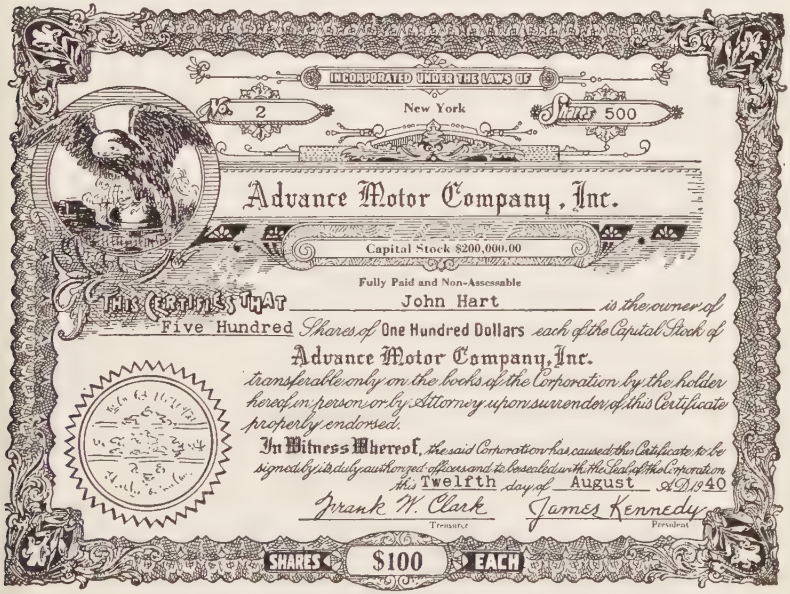
Kennedy & Hart decided to put their new motor on the market, but to do so they required more money than they had. It was necessary to interest others in the new motor, and James approached a number of successful business men, who agreed to invest money in the project.

A new business organization, called a **corporation,** was formed under the name ADVANCE MOTOR COMPANY, INC. To form the corporation, a **charter,** or license, had to be obtained from the state permitting the corporation to do business and to issue and sell shares of **capital stock** in the company.

The Advance Motor Company issued 2000 shares of capital stock having a stated value, or **par value,** of $100 a share. The total capital stock of the company was therefore $200,000.

As owners of the invention and in return for the money they put into the business, James and John each received

500 shares of the Advance Motor Company stock. The men who had agreed to invest money in the project bought a total of 600 shares, and the rest of the shares were sold to others. Each **stockholder,** or person who bought stock, became a part owner in the business and received a **stock certificate** like the one below, giving the number of shares he owned and the par value of each share.


INCORPORATED UNDER THE LAWS OF
New York
No. 2
Shares 500
Advance Motor Company, Inc.
Capital Stock $200,000.00
Fully Paid and Non-Assessable
THIS CERTIFIES THAT John Hart is the owner of
Five Hundred Shares of One Hundred Dollars each of the Capital Stock of
Advance Motor Company, Inc.
transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.
In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation this Twelfth day of August A.D. 1940
Frank W. Clark
Treasurer
James Kennedy
President
SHARES $100 EACH

A STOCK CERTIFICATE

The stockholders of the Advance Motor Company met and elected **directors** for the coming year, each stockholder having as many votes as he owned shares of stock. The directors elected James Kennedy and John Hart as president and vice-president and also elected a treasurer and other officers of the corporation. The directors were to decide the policies of the company and distribute part of any profits to the stockholders as **dividends.**

At the end of the first year, the Advance Motor Company had made a profit of \$18,680. The directors decided to divide \$12,000 of the profit among the stockholders as dividends and to hold the balance, or \$6680, as **surplus.**

The capital stock of the corporation was \$200,000, and the amount to be divided was \$12,000. The rate of dividend **declared,** or agreed to be paid, was therefore \$12,000 ÷ \$200,000, or 6% of the par value. Since the par value of each share was \$100, a dividend of 6% meant \$6 a share.

1. James Kennedy and John Hart each had 500 shares of Advance Motor Company stock.

a. What was the total par value of the shares each one held in the company?

b. What amount did each receive as his first dividend?

c. What per cent of the entire dividend did each receive?

If the shares of a corporation have a par value, dividends are usually a certain per cent of the par value; for example, 5% or 6%. The shares of some corporations have no par value, in which case the dividends are a definite amount per share; for example, \$4.75 per share.

PROBLEMS IN STOCKS

1. A company's capital stock is \$850,000, and the par value of each share is \$100. How many shares of stock can be issued by the company?

2. A company having a capital stock of \$125,000 paid a dividend of \$6250. What was the rate of dividend?

3. Find the dividend on 35 shares of stock on which a dividend of \$4.75 per share is paid.

4. A dividend of $5\frac{1}{2}\%$ is declared on stock having a par value of \$100. What is the dividend on 15 shares?

Some corporations issue two kinds of stock, **preferred** and **common.** Usually, a fixed rate of dividend is paid on preferred stock, provided the corporation prospers and a dividend is declared. The dividend paid on common stock depends on the amount of profit left after the preferred stock dividend has been paid. If the profits have been large, the common stock may pay a higher dividend than the preferred stock. If the profits have been small, no dividend may be paid on either stock.

Many people invest in stocks for the return which they get in dividends. In buying stock for investment, three points should always be kept in mind:

1. **Will the money invested be safe? Is the company which issued the stock sound?**
2. **Can the stock be expected to pay a fair dividend?**
3. **Is there a ready market for the stock?**

Shares of stock may be bought and sold like any other kind of property. Most purchases and sales of stocks are made at places called **stock exchanges,** the most important in the United States being the New York Stock Exchange in New York City. Men who buy and sell stocks for others are known as **brokers,** and the commission they receive is called **brokerage.** The brokerage rates charged on the New York Stock Exchange range from 1¢ to more than 22¢ a share, depending upon the price at which the stock is bought or sold.

The **market value,** or price at which a stock can be bought or sold, is usually higher or lower than the par value of the stock. If a company is prosperous and pays regular dividends, its stock may sell for more than the par value, or **above par.** If the dividends are irregular or small, the stock will sell **below par.**

The prices at which stocks sell on the New York Stock Exchange are usually the standard prices throughout the country and are printed daily in all large newspapers. These prices, called **stock quotations,** are given in dollars and fractions of a dollar.

Below are listed some of a day's stock sales quoted from a newspaper. Reading across, the table gives: (1) the number of hundreds of shares of stock sold; (2) the name of the stock and the number of dollars which it last paid in annual dividends; (3) the price per share when the exchange opened; (4) the highest price during the day; (5) the lowest price for the day; and (6) the price when the exchange closed.

NEW YORK STOCK EXCHANGE QUOTATIONS

Sales in 100's	Stock and Dividend in Dollars	Open	High	Low	Close
27	Am Tel & Tel 9	165	$165\frac{3}{4}$	$163\frac{3}{4}$	$165\frac{3}{4}$
289	Gen Elec .95	$40\frac{7}{8}$	$41\frac{1}{4}$	$40\frac{1}{4}$	$41\frac{1}{4}$
7	Norfolk & Western 10	$188\frac{1}{2}$	$197\frac{1}{2}$	$188\frac{1}{2}$	$197\frac{1}{2}$
138	US Steel	$70\frac{7}{8}$	$71\frac{1}{2}$	70	$71\frac{1}{2}$
3	US Steel pf 7	$117\frac{1}{2}$	$117\frac{3}{4}$	$117\frac{1}{2}$	$117\frac{1}{2}$

The first quotation reads: Sold 2700 shares of American Telephone and Telegraph Company stock; last yearly dividend, \$9 a share; opening price, \$165; high for the day, \$165.75; low for the day, \$163.75; closing price, \$165.75.

1. Read the other quotations in the table. The abbreviation **pf** stands for **preferred stock.**

2. Bring to class a stock market page from a newspaper. Choose several stocks, and tell the sales, the high, the low, and the opening and closing prices of each.

The federal government and some state governments charge a **stock transfer tax** on all stock transactions, the tax being based on the price of the stock.

How to Find the Total Cost of a Purchase of Stock

What was the total cost of 100 shares of stock bought at $16\frac{3}{4}$, if brokerage was 14¢ per share?

\$16.75 × 100 = \$1675 cost of stock	Multiply the price per share by the number of shares.
\$.14 × 100 = \$14 brokerage	Multiply the brokerage per share by the number of shares.
\$1675 + \$14 = \$1689 total cost of stock	To find the total cost, add the brokerage to the cost of the stock.

To find the *proceeds* when *selling* stock, find the amount for which the shares were sold and *subtract* the brokerage.

Find the total cost of each of these stock purchases:

	No. of Shares	Purchase Price	Brokerage Per Share		No. of Shares	Purchase Price	Brokerage Per Share
1.	20	$119\frac{1}{4}$	25¢	**4.**	700	$132\frac{1}{8}$	26¢
2.	300	$88\frac{1}{2}$	21¢	**5.**	30	$51\frac{7}{8}$	19¢
3.	50	$18\frac{5}{8}$	15¢	**6.**	400	$35\frac{1}{4}$	16¢

Find the proceeds of each of these sales of stock:

	No. of Shares	Selling Price	Brokerage Per Share		No. of Shares	Selling Price	Brokerage Per Share
7.	200	113	24¢	**10.**	60	$91\frac{3}{8}$	23¢
8.	40	$56\frac{3}{4}$	19¢	**11.**	700	$115\frac{1}{4}$	24¢
9.	800	$18\frac{1}{8}$	14¢	**12.**	300	$109\frac{3}{8}$	23¢

13. Mr. Martin purchased 40 shares of stock at $156\frac{1}{8}$. What was the total cost, including brokerage of 29¢ a share?

14. What proceeds did Mr. Lucas receive from the sale of 700 shares of stock at $73\frac{5}{8}$; brokerage, 20¢ a share?

Although stocks are often sold above or below par, the dividends paid are always figured on the par value of the stock. On a stock having a par value of \$100 and paying a 5% dividend, the dividend would be \$5 a share, **no matter what price was paid for the stock.** In order to find his actual rate of income from a stock investment, the purchaser must find what per cent the dividend is of the purchase price.

How to Find the Rate of Income from a Stock

Mr. Walton bought at 124 some stock which had a par value of \$100. The stock paid a 6% dividend. What was his rate of income from the investment?

6% of \$100 = \$6 a share dividend	Find the dividend on one share.
\$6 ÷ \$124 = .0483+ = 4.8% rate of income	To find the rate of income, divide the dividend on one share by the cost per share.

Remember: Rate of income on stock = dividend ÷ cost price.

In finding the rate of income, brokerage and other expenses of purchase are not usually considered, because they are so small.

If a dividend is paid quarterly or semiannually, use the total dividend for the year to find the rate of income.

Find the rate of income from each stock, par value \$100:

	Cost	Annual Dividend		Cost	Annual Dividend		Cost	Quarterly Dividend
1.	80	4%	3.	$62\frac{1}{2}$	4%	5.	184	\$2.25
2.	150	6%	4.	156	8%	6.	45	\$.85

7. Which gives the higher rate of income: 8% stock at 160, or 5% stock at 92? (Par value of each, \$100.)

After consulting with his banker, Mr. Andrews bought stock in a company which, though young, was well managed and conservative. He knew that stock often increases in value if the company which issues it is ably directed and can pay regular dividends. Some years later, Mr. Andrews was able to sell the stock at a considerable profit because he had invested wisely and had been willing to wait until his investment increased in value.

Many people buy stock with the idea of selling it later at a higher price, as Mr. Andrews did. However, unless they use good judgment and forethought and ask competent advice, they may lose instead of gain.

Mr. Eastman was anxious to make a quick profit. He knew that the market value of an unsteady stock often changes greatly from day to day, so he bought such a stock in the hope of selling it within a few days at a profit. Instead of rising, the price of his stock fell; and Mr. Eastman lost money. He had not been willing to wait for a growth in the value of a sound stock, but had **speculated,** or taken a chance on a quick profit—and had lost. Speculation frequently results in heavy losses. **Do not speculate!**

Find the profit or the loss in each of the following. The profit or the loss is the difference between the total cost of the stock and the proceeds from the sale of the stock, including brokerage in each case.

	Number of Shares	Purchase Price	Purchase Brokerage	Selling Price	Selling Brokerage
1.	100	88	21¢	$96\frac{1}{4}$	22¢
2.	200	$62\frac{1}{8}$	19¢	$48\frac{1}{2}$	17¢
3.	700	$106\frac{1}{4}$	23¢	$111\frac{3}{8}$	24¢

4. Mr. Graham bought 300 shares of stock at $112\frac{1}{2}$ and later sold them at $116\frac{5}{8}$. Brokerage was 24¢ a share in each case. What was his profit on the sale of the stock?

1. What dividend will I receive on 40 shares of stock, par value \$100 a share, if the dividend rate is $4\frac{1}{4}\%$?

2. I bought 20 shares of mining stock at $26\frac{1}{8}$. The brokerage charge was 16¢ a share. How much did the shares cost?

3. I received a quarterly dividend of \$1.50 a share on stock which I had purchased at 160. What was the rate of income from my investment?

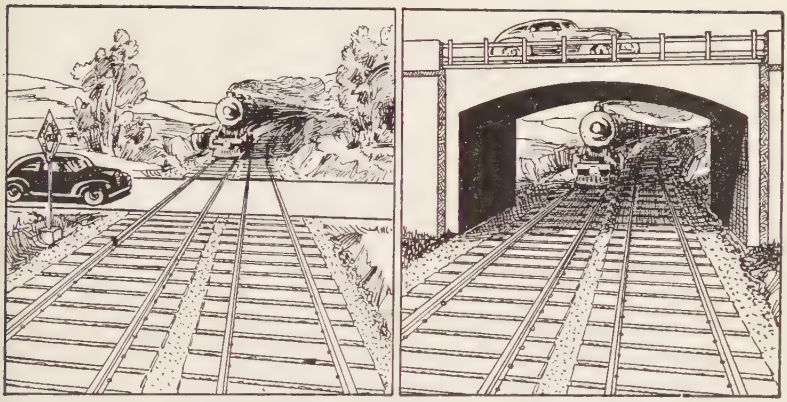
4. Mr. Porter noticed that the price of a certain oil stock had risen within a few days. He bought 200 shares at $34\frac{1}{8}$, hoping to sell soon at a profit. However, the price fell, and Mr. Porter sold his stock two days later at $32\frac{3}{4}$. Brokerage was 16¢ a share in each case. How much did he lose?

5. A certain stock paying a 5% dividend sells at 120. Another stock, paying a quarterly dividend of \$1.40 a share, sells at 130. Par value of both stocks is \$100. Which stock offers the greater rate of income? How much greater?

6. The 5% preferred stock of a company is selling at 92. The common stock, on which an 8% dividend has just been declared, is selling at 120. Par value of both stocks is \$100. Which stock yields the greater rate of income? If business conditions are uncertain, which stock is the safer investment? Why?

7. Mr. Allen bought at 90 a share 50 shares of stock having a par value of \$100. Brokerage was 23¢ a share. At the end of a year, after receiving a 5% dividend, he sold the stock at 110. Brokerage on the sale was 24¢ a share. Find his total profit, including the dividend.

8. Select four stocks listed in a newspaper. Find the cost of 200 shares of each at the "low" price. Find the amount of a sale of 200 shares of each at the "high" price. In each case, disregard brokerage.



Money raised from the sale of state bonds helps to pay for the removal of dangerous grade crossings.

Corporations and governments often need large sums of money for special purposes. A manufacturing company needs money to build a modern factory. A railroad company needs funds to electrify its lines or improve its roadbed. A city or town must build new schools or provide a new water system. A state needs money to remove grade crossings or to construct a state hospital. The federal government must have money to pay for some national project, such as a ship canal. To raise the money needed, the corporation or government may issue **bonds.**

A *bond* is a form of note which is a promise to repay borrowed money on a certain date and to pay interest on the money at a given rate.

Bonds are usually issued in amounts of $1000, $500, or $100, and run for terms varying from 5 to more than 50 years. The date when a bond matures, or must be paid, together with the amount of interest to be paid and the dates of interest payments are stated in the bond. Interest on bonds is usually payable semiannually, and is based on the par value, or amount for which the bond is written.

Bonds issued by corporations are usually secured by real estate or some other form of property. If a corporation fails to pay its bonds when they mature, or fall due, it must turn over to the bondholders the property it pledged as security. Government bonds are protected by laws which limit the amount a government may borrow. This prevents the government from borrowing so much that payment is doubtful.

Two forms of bonds are issued, **registered bonds** and **coupon bonds.** Registered bonds bear the names of the persons to whom the bonds are sold. The corporation or government which issues registered bonds keeps a record of the purchasers and mails to them the interest checks when due. When a registered bond is sold, the record is changed to show the name of the new owner.

Coupon bonds have attached to them a series of interest coupons, one for each interest payment. Each coupon states the amount of interest and the date it will become due. When a coupon becomes due, the owner cuts it off the bond and presents it at a bank for payment or collection.

Like stocks, bonds are bought and sold on stock exchanges. Their selling prices, or market values, are printed in the newspapers. Bonds may sell above or below par.

The prices of bonds are quoted on a basis of \$100 par value. The quotation **NY Cent $3\frac{3}{4}$s '46 80** means that New York Central Railroad bonds paying $3\frac{3}{4}\%$ interest, maturing in 1946, are selling at \$80 per \$100 of par value. The price of a New York Central \$500 bond would be $5 \times \$80$; the price of a \$1000 bond would be $10 \times \$80$.

United States government bonds are quoted to thirty-seconds of a dollar. A government bond quotation of 105.8 means $\$105\frac{8}{32}$, or \$105.25. A \$1000 government bond at this price would cost $10 \times \$105.25$, or \$1052.50.

Turn to the bond quotations in a newspaper. Choose several bonds listed, and tell what each quotation means.

How to Find the Total Cost of a Purchase of Bonds

Mr. Martin bought six \$1000 bonds at $97\frac{3}{8}$ plus brokerage of \$2.50 per bond. What was the total cost of the bonds?

\$97.375 × 10 = \$973.75 price of one bond	Find the price of one bond.
\$973.75 × 6 = \$5842.50 cost of six bonds	Multiply the price of one bond by the number of bonds.
\$2.50 × 6 = \$15 brokerage	Find the brokerage.
\$5842.50 + \$15 = \$5857.50 total cost of bonds	To find the total cost, add the brokerage to the cost of the bonds.

To find the *proceeds* from the *sale* of bonds, subtract the brokerage from the selling price.

Find (a) the total cost of each of these purchases of bonds and (b) the amount of interest received semiannually from the investment.

. One 4% \$1000 bond at 88, brokerage \$2.50

. Ten \$1000 NYC RR $4\frac{1}{2}$s at $61\frac{7}{8}$, brokerage \$2.50 each

. Six \$1000 U.S.Treasury $3\frac{3}{8}$s at \$102.28, brokerage \$2.50 each

. Fifteen $3\frac{1}{4}$% \$500 bonds at $102\frac{1}{8}$, brokerage \$1.25 each

Find the proceeds from these sales of bonds:

. Four \$1000 4% bonds at $68\frac{3}{4}$, brokerage \$2.50 each

. Eight \$1000 $3\frac{1}{4}$% bonds at $105\frac{1}{2}$, brokerage \$2.50 each

. Twenty-five \$500 $5\frac{1}{2}$s at $72\frac{5}{8}$, brokerage \$1.25 each

The price paid for a bond has no effect on the amount of interest it pays, since interest is figured on the par value. However, the price paid does affect the **yield,** or rate of income, from the bond.

How to Find the Rate of Income from a Bond

Mr. Sheridan bought a 6% $1000 bond at 112. What was his rate of income from the bond?

$.06 \times \$1000 = \$60$ yearly interest — Find the yearly interest on the bond.

$10 \times \$112 = \$1120$ purchase price — Find the purchase price of the bond.

$\$60 \div \$1120 = .0535+ = 5.4\%$ rate of income — To find the rate of income, divide the yearly interest by the purchase price of the bond.

In finding the rate of income from a bond, brokerage is usually omitted.

Find the rate of income from each of the following bonds:

	Par Value	Interest	Purchase Price
1.	$1000	5%	82
2.	$1000	$4\frac{1}{2}\%$	$104\frac{3}{4}$
3.	$1000	5%	$118\frac{1}{2}$
4.	$500	$3\frac{1}{2}\%$	105
5.	$100	4%	$108\frac{1}{2}$
6.	$500	5%	84

7. Mr. Carlson owns a United States Treasury bond which he bought at 104.24. The bond pays $3\frac{1}{4}\%$ interest, and its par value is $1000. Find the rate of income from the bond.

8. Mr. Jackson can invest in either of these two $1000 corporation bonds: bond A, paying 4% interest and selling at 90; or bond B, paying 5% interest and selling at 105. Which bond would give him the higher yield? About how much higher?

The United States government offers a practical way for the man with an average income to provide funds for the future through the purchase of **United States Savings Bonds.** The cost of these bonds, which mature in 10 years, is:

\$18.75 for a \$25 bond	\$75 for a \$100 bond
\$37.50 for a \$50 bond	\$375 for a \$500 bond
\$750 for a \$1000 bond	

This means that money invested in United States Savings Bonds will increase $\frac{1}{3}$ or $33\frac{1}{3}\%$ in 10 years.

Suppose a man 50 years old with a good job buys a Savings Bond for \$37.50 each month for 10 years. When he is 60, the first of those bonds will mature; and each month until he is 70 he will receive \$50 from the United States government. In this way, by investing \$450 a year for 10 years while he is still able to earn a good salary, he is assured an income of \$600 a year between the ages of 60 and 70, when his earnings may not be so high.

1. Mr. Jordan bought 4 Savings Bonds for \$75 each, and 2 for \$375 each. How much will he receive from the government when the bonds mature? How much more will he receive than he invested?

Savings Bonds cannot be sold or transferred, but they may be cashed at any time after 60 days from the date of purchase. Get from your local post office, where Savings Bonds are sold, a card showing the cash value of these bonds after they have been held for different lengths of time. Study the card, then answer the following questions:

2. What would be the cash value of six \$100 bonds held for $6\frac{1}{4}$ years?

3. What would I receive for three \$1000 bonds cashed $2\frac{1}{2}$ years after I purchased them?

1. A city borrowed \$265,000 by selling 5% bonds due in 25 yr. How much was the total interest paid on the bonds (a) each year; (b) during the 25 years?

2. To meet an emergency, Mr. Wilson cashed a \$500 U. S. Savings Bond. The bond had run $2\frac{1}{4}$ yr. and was worth \$19.50 for each \$25 of par value. How much did Mr. Wilson receive for the bond?

3. A man owns 17 bonds having a par value of \$500 and paying $4\frac{1}{2}\%$ interest, payable semiannually. How much interest does he receive at the end of each six months?

4. Mr. Walton bought six \$1000 6% bonds at $112\frac{1}{2}$. Brokerage was \$2.50 per bond. How much did the bonds cost him?

5. What is the yield from a \$1000 bond paying 6% and bought at 96?

6. Which of these \$1000 government bonds would give the greater yield: a 3% bond bought at 105.8, or a 4% bond bought at 109.24?

7. Mr. Thorne owned the following \$1000 bonds: four $3\frac{1}{2}\%$ government bonds, three 5% city bonds, and five 6% corporation bonds. (a) How much was his total yearly income from the bonds? (b) If the total cost of the bonds was \$12,120, what was the total annual yield?

8. Four \$100 5% bonds were purchased at $94\frac{3}{8}$ plus brokerage of \$1 per bond. (a) What was the total cost of the bonds? (b) What was the annual income? (c) What per cent of the total cost was the annual income?

9. Mr. Connolly bought five 5% \$1000 bonds at $104\frac{3}{4}$. After receiving one year's interest, he sold the bonds at 108. Brokerage on both the purchase and the sale was \$2.50 per bond. What was his total profit, including interest?

1. United States government bonds and the bonds of outstanding corporations pay a lower rate of interest than most other bonds. Because of their greater safety, high-grade bonds sell at high prices. Bonds that are less safe usually bear higher rates of interest and sell at lower prices. Why?

2. Bonds, in general, are a safer investment than stocks because they are secured by real estate or in other ways.

3. The interest on the bonds of a company must be paid before any dividends can be paid on the stock.

THE POSTAL SAVINGS SYSTEM

To encourage young people and people with small incomes to save, the United States government has established the **Postal Savings System.** This system is operated by the Post Office Department. Anyone 10 years old or over may open a savings account at one of the United States post offices and may deposit any number of dollars from $1 to $2500. Simple interest at the rate of 2% a year is paid on all accounts. The United States government is solemnly pledged to protect the postal savings accounts and to pay interest on them.

Postal-savings certificates, which are issued in various denominations, or amounts, from $1 to $500, are made out in the name of the depositor for the amounts he deposits, and serve as his receipts for the money deposited.

Although no deposits of less than $1 are accepted, smaller amounts may be saved by buying postal-savings stamps at 10¢ each. Cards on which 10 of these stamps may be pasted are furnished by post offices. A full card will be accepted as a deposit of $1, or will be redeemed in cash.

1. From your post office, get a circular describing the Postal Savings System, and discuss it in class.

Sometimes, the owner of real estate wants to borrow a sum of money and asks an acquaintance or a bank to lend it to him. In return for the money, he gives the lender two papers. One is a **bond,** or note, containing a promise to repay the money at a stated time and to pay interest at a certain rate. The other is a **mortgage,** or claim against the real estate for the amount of the loan with interest, in case the money is not paid when due. Some mortgages call for part payments on the principal at stated times. If the borrower fails to pay the principal or the interest when due, the holder of the mortgage may **foreclose** — that is, after a certain period he may ask the courts to sell the real estate and to repay him from the proceeds of the sale.

As a rule, real estate is not mortgaged for more than 50% or 60% of its value. Moreover, a property owner cannot borrow money on his real estate unless he has a clear **title,** or proof of ownership, to the property and all taxes on it have been paid. Also, he must insure the buildings in favor of the lender to protect him in case the buildings should burn.

Because the holder of a mortgage is so well protected against loss, well-selected mortgages are generally considered good investments. They usually pay higher rates of interest than other safe investments.

Find the annual interest and the total interest on each of these mortgage loans:

	Mortgage	Rate	Term		Mortgage	Rate	Term
1.	$4000	6%	4 yr.	4.	$2400	$5\frac{1}{2}$%	16 yr.
2.	$3500	5%	6 yr.	5.	$1680	6%	7 yr.
3.	$2800	$4\frac{1}{2}$%	12 yr.	6.	$6400	$5\frac{1}{4}$%	13 yr.

7. Learn what you can about **first** and **second mortgages.**

8. Find out why a mortgage must be recorded in the county clerk's office when made. Find out how its payment should be recorded.

To show that you know the meaning of each of the following terms, (1) define it, or (2) use it correctly in a sentence.

1. Mature
2. Bond
3. Broker
4. Invest
5. Directors
6. Dividend
7. Par value
8. Registered bond
9. Stock quotation
10. Stock exchange
11. Rate of income
12. Preferred stock
13. Stock transfer tax
14. Stock certificate
15. Foreclose
16. Coupon bond
17. Stockholders
18. Common stock
19. Corporation
20. Surplus
21. Capital stock

Complete each of the following statements by using the word or expression in parentheses that makes the statement true:

22. The fee paid to a person for buying or selling stocks or bonds is usually called (dividend—brokerage—commission).

23. A written pledge of real estate as security for the payment of a loan is a (premium—note—mortgage).

24. The rate of income from a bond is the (quotation—dividend—yield).

25. A license which permits a corporation to do business and to issue stock is a (stock certificate—charter—share).

26. The price at which stocks or bonds can be bought or sold is the (par value—proceeds—market value).

27. Stocks or bonds which sell for more than par value are said to sell (above par—at par—below par).

28. Buying stock on the chance of a quick profit is known as (investment—speculation—foreclosing).

Try to work these problems without using pencil and paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. By investing $28 in a bicycle, Larry was able to get a job delivering packages for a druggist. If he earned enough in 8 weeks to pay for his investment, what were his average weekly earnings?

2. John took a course in business training at a night school. As a result, his employer raised his pay from $65 to $85 a month. How much additional salary did his investment in study earn for him in 1 year?

3. A corporation had a capital stock of $60,000. How many shares of stock, par value $50, could it issue?

4. On a 5% mortgage of $3200, interest is paid semi-annually. How much interest is paid each 6 mo.?

5. At 17¢ a share, what is the brokerage on 200 shares of stock?

6. A man sold sixty $100 bonds at 120. How much did he receive for the bonds, not including brokerage?

7. Mr. Yates invested $500 in the stock of a large corporation. His dividends for the first year were $22.50. What was his rate of income from the investment?

8. Mr. Jennings bought 12 shares of railroad stock at 87 and sold them at 94. What was his profit, not including brokerage?

9. If a bond having a par value of $100 and paying $4 annual interest is bought at 80, what is the yield from the investment?

Work the examples in column A. If you fail on any example, do the example opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review

1. Finding the Total Cost of a Purchase of Stock

No. of Shares	Purchase Price	Brokerage Per Share	No. of Shares	Purchase Price	Brokerage Per Share	
300	$112\frac{1}{2}$	24¢	500	169	29¢	471
400	$12\frac{3}{4}$	14¢	700	86	21¢	

2. Finding the Proceeds of a Sale of Stock

No. of Shares	Selling Price	Brokerage Per Share	No. of Shares	Selling Price	Brokerage Per Share	
300	$72\frac{1}{2}$	20¢	200	$96\frac{1}{4}$	22¢	471
800	42	17¢	600	$105\frac{1}{4}$	23¢	

3. Finding the Rate of Income from a Stock

Purchase Price	Annual Dividend	Par Value	Purchase Price	Annual Dividend	Par Value	
162	6%	$100	148	4%	$100	472
$68\frac{1}{4}$	$2.10	none	$72\frac{1}{2}$	$3.50	none	
$28\frac{1}{4}$	$2\frac{1}{2}$%	$25	$22\frac{1}{2}$	$3\frac{1}{2}$%	$25	

4. Finding the Total Cost of a Purchase of Bonds

Eight $500 bonds Bought at $103\frac{3}{8}$ Brokerage, $1.25 ea.	Six $1000 bonds Bought at $108\frac{1}{8}$ Brokerage, $2.50 ea.	477

5. Finding the Yield, or Rate of Income, from a Bond

Par Value	Interest	Purchase Price	Par Value	Interest	Purchase Price	
$1000	4%	86	$1000	5%	106	478
$500	$5\frac{1}{2}$%	110	$500	$4\frac{1}{2}$%	88	
$100	$4\frac{3}{4}$%	$107\frac{5}{8}$	$100	$5\frac{3}{4}$%	$94\frac{3}{8}$	

Explain how you would find the answer to each of the following. Show you are right by making a problem with reasonable numbers and working and checking it.

1. You know how many times you were at bat and how many hits you made. How would you find your batting average?

2. You know the amount of life insurance you wish to take out and the premium rate per $1000. How would you find the premium?

3. A commission merchant sells a shipment of tomatoes for you and charges commission and trucking expenses. How can you find the proceeds you should receive?

4. You know the selling price of a washing machine and the per cents allowed for profit and overhead, based on the selling price. How would you find the profit? The cost?

5. You know the assessed valuation of your property and the tax rate per $1000. How can you find the amount of your tax?

6. You sell your automobile, receiving part of the payment in cash, and a note with interest for the balance. If a bank discounts the note for you a few days later, how can you find the proceeds of the note?

7. If you insure a house against fire loss for part of its value, how can you find the cost of insurance if the rate per $100 is given?

8. If you know the cash surrender rate per $1000 on your life insurance policy, how can you find the cash surrender value?

1. A strip of canvas 15 yd. 1 ft. long is divided into 6 equal lengths. What is the length of each piece?

2. A traveling salesman drives his car 30,000 mi. a year. If he pays 4¢ tax on each gallon of gasoline he uses and his car travels 15 mi. to the gallon, find: (a) the number of gallons of gasoline he uses in 1 yr.; (b) the amount of gasoline tax he will pay during 1 yr.

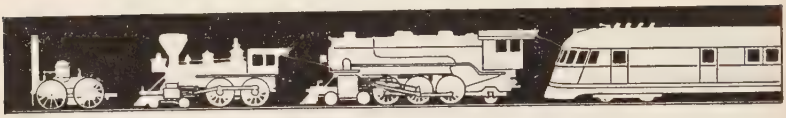
3. Find the interest on \$818 at 6% for 99 da., using the 60-day method. Use another method to check your result.

4. At what price must Mr. Todd sell chairs costing \$16.25 each if he wishes to allow 20% for overhead and 15% for profit, based on the selling price?

5. To meet school expenses, \$20,607 must be raised by taxing property which has an assessed valuation of \$1,648,560. (a) What will be the tax rate per \$1000? (b) Express the tax rate in three other ways.

6. Mr. Gray took out a 30-yr. endowment life insurance policy for \$8000. The yearly rate was \$32.10 per \$1000. (a) How much was his yearly premium? (b) What will be the total amount paid in premiums if he lives until the policy becomes due? (c) When the policy becomes due, how much more will he collect than he paid in premiums? Disregard dividends.

7. Make out a receipted invoice for the following order: On June 8, John Hall, 16 Blank St., Hartford, Conn., bought from the Northern Supply Co., Buffalo, N.Y., 2 tires @ \$11.75; 6 auto lamp bulbs @ 12¢; 4 brake bands @ 90¢; and 2 hammers @ \$1.18. Terms were 2%, 10 da.; net, 30. The invoice was paid on June 15.



1. Add 7.145, 36.05, 471.5, .368, and 5.1.

2. Add 1 yd. 2 ft. 7 in. and 3 yd. 1 ft. 10 in.

3. Find the difference between 12.35 and 4.625.

4. $1\frac{3}{5} \times 1\frac{1}{4} \times \frac{5}{8} = ?$

5. $4\frac{3}{4} \div 1\frac{3}{4} = ?$

6. $2\frac{1}{4}\%$ of $360 = ?$

7. $2 \div .25 = ?$

8. Divide the product of 360.5 and .06 by 1.24.

9. A company has a capital stock of $650,000, with a par value of $25 per share. What is the number of shares?

10. How much is each interest payment on a mortgage loan of $2725 at $5\frac{1}{4}\%$, payable quarterly?

11. How much would six $1000 bonds cost at $98\frac{3}{8}$, with brokerage of $2.50 per bond?

12. What is the rate of income from stock having a par value of $100 and bought at 106 if the yearly dividend is 5%?

13. What proceeds did Mr. Todd receive from the sale of seven $500 bonds sold at $101\frac{7}{8}$, with brokerage at $1.25 per bond?

14. Find the yield of a $3\frac{1}{2}\%$ $1000 bond which is bought at $97\frac{1}{4}$.

15. Mr. Colt bought 200 shares of stock at $165\frac{1}{8}$, plus brokerage of 29¢ a share. Two years later, he sold the stock at $169\frac{7}{8}$, with brokerage of 29¢ a share. How much was his profit on the stock?

*Record your score on your Progress Chart. See page 321.

One night, the Breen family were all at home. Suddenly George said, "Say, isn't there something we can do?"

"Yes," said his mother. "We can play an arithmetic game, and I'll offer a prize to the winner. Each one will tell how he has used arithmetic lately and will give the rest of us the problems he had to work. I will give a box of candy to the one who makes the best problems. Caroline, you begin."

Caroline thought a few minutes and then gave these problems. Work them.

1. I went shopping this morning at the Lincoln Store and bought $1\frac{1}{8}$ yd. of velvet @ \$1.98; $2\frac{3}{4}$ yd. of woolen cloth @ \$1.44; and $1\frac{7}{8}$ yd. of silk @ \$1.76. I paid for the goods and brought them home. Make out my sales slip, show the amount paid, and tell what change I received from \$10.

2. I have agreed to buy the wool and to knit a sweater for Aunt Mary for \$15. It will take 6 oz. of wool at 80¢ an ounce. If I knit 4 hours a day, it will take me 17 days to finish it. How much an hour will I get for my time?

When everyone had worked Caroline's problems, George gave his. Work them.

3. My report cards show these marks for the last 3 mo.: History—77, 83, 80; English—90, 95, 92; Arithmetic—87, 94, 90. I made a line graph comparing these sets of marks. Make the graph.

4. I have been selling Easter cards for a dealer and receiving 20% commission. Yesterday, I sold 3 boxes of cards @ 75¢, 2 boxes @ $1.25, and 12 cards @ 5¢ each. Find my commission and the net proceeds to the dealer.

5. This is the 30th of the month. On the 1st, I had $3.50 cash on hand. Since then, I have received my allowance of 50¢ on the 1st, 8th, 15th, 22nd, and 29th, and have earned $2.24 commission. So far this month, I have spent 15¢ for candy, $1.49 for shirts, 75¢ for a book, and 40¢ for a movie. Make out and balance my cash account, supplying necessary dates.

6. A notice came this morning saying that if I renew my subscription to the *Science Magazine* at once, I can renew it for $2.45 instead of $3.50, the regular price. What per cent will I save by accepting the offer?

"Guess that will do for me, Mother," said George. "Now it's your turn."

Mother began: "As I am the buyer for this family, I shall try to prove that I buy wisely by making each dollar spent go as far as possible."

7. Our gas bill this month was $3.65. By paying it within 10 days, I got a discount of 3%. How much did I save?

8. I bought 20 lb. of sugar today at 10 lb. for 54¢. How much did I save over the regular price of 6¢ a lb.? How much was the saving on each pound?

9. The regular price of corn is 2 cans for 25¢. Today, I bought a case of 24 cans for $2.28. How much did I save by buying by the case? What was the saving on each can?

10. Hunter's Store had a sale today, and I bought a dozen 35¢ towels at 25% off; a floor brush marked $1.35 with 27¢ off; 3 pairs of 79¢ stockings at 3 pairs for $2.25; and a house dress marked $2.49 for $\frac{2}{3}$ of its price. How much did I spend, and how much did I save by buying at the sale?

11. My household account has four column headings: Date, Items, Received, Paid. The first of this week, I had $12.67 cash on hand, and your father gave me $20. During the week, I spent $2.64 for meat, 91¢ for milk, $6.17 for other groceries, 30¢ for the newspaper, and $8.14 for miscellaneous items. Make out and balance my account for the week, supplying necessary dates.

12. Here is a bill for last month. Check it for errors, and tell me how much I owe the Caldwell Company.

CALDWELL FURNITURE COMPANY
SALEM, OREGON

Feb. 1, 19__

Sold to Mrs. C. H. Breen
Malden Apartments

Date	Items	Charges		Credits		Last Am't Is Balance Due	
Dec. 31	Balance due					14	50
Jan. 5	6 dining room chairs @ 6.18	37	18			51	68
8	Cash			14	50	37	18
12	4½ yd. chair covering @ 2.05	9	23			46	41
24	Electric refrigerator	94	15			140	56

"Now, Father, it's your turn," said Mother.

"Well, here goes," said Father.

13. Breen & Company's expenses have been heavy this month, and we had to borrow \$5500 from the City Bank to help pay cash for some goods, giving our note for 90 days.

a. The bank discounted our note at 6%. What proceeds did we receive?

b. By paying cash for the goods, we received a 2% discount from the net amount of the bill. The bill less the discount was \$5635. What was the amount of the bill before the discount was taken? How much did we save by borrowing money so we could pay cash?

c. We figure that our overhead on the goods, which cost \$5635, will be \$1268. For how much must we sell the goods to make a profit of 8% of the cost?

14. The other day, I figured up what our car had cost us last year. Here are the items charged against the car. Copy the list, and fill in the amounts which are lacking. Then find the total cost for the year and the average cost per month.

Depreciation. 25% of \$880, cost of the car . .	\$____
Insurance	68.50
Drivers' licenses. 50¢ each for two drivers . .	____
Automobile license. 50¢ per 100 lb. (The automobile weighs 2700 lb.)	____
Repairs	18.40
Spare tire	10.50
Gasoline. 648 gal. at an average cost of $16\frac{1}{2}$¢ per gallon	____
Oil. 53 qt. @ 25¢ a quart	____
Miscellaneous	3.25
Total annual cost	\$____
Average monthly cost . . .	\$____

To which member of the Breen family would you give the prize box of candy? Why?

A plan for using an income is called a *budget*. A plan for using a family income is called a *family budget*.

The purpose of a family budget is to balance family income and expense and allow for saving.

In a well-planned family budget, the expenses are usually grouped under the following headings:

Food	Advancement (education,	Other Expenses
Clothing	recreation, charity, church)	Savings
Shelter	Operating Expenses	

Tell under which of the above headings you would enter the following items, and why.

1. Rent	**5.** Coal	**9.** Doctor's bill
2. Telephone	**6.** Lecture tickets	**10.** Payment on car
3. Groceries	**7.** Savings deposit	**11.** Dress goods
4. New coat	**8.** Furnace repairs	**12.** Church

The Stewart's Monthly Budget	
Food	$50
Clothing	30
Shelter	50
Operating Expenses	20
Advancement	16
Other Expenses	14
Savings	20
Total	$200

13. The way the Stewart family budgets its monthly income is shown at the left. Find what per cent of the monthly income is allowed for each item.

14. Mr. Emory earns $40 a week. He budgets his weekly income as follows: food, 25%; clothing, 14%; shelter, 24%; operating expenses, 10%; advancement, 8%; other expenses, 11%; savings, the balance. Show this budget in dollars and cents.

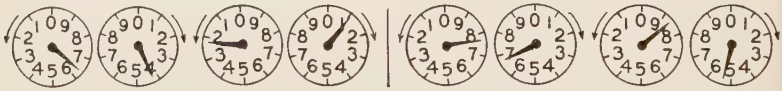
15. If emergencies cause extra expenditures in one month, what must be done to keep within the yearly budget?

16. Give the advantages of keeping a family budget.

Electric power is measured by the **watt (w.)** The amount of work done by one watt in 1 hr. is a **watt-hour.**

1000 watt-hours (w-hr.) = 1 kilowatt-hour (kw-hr)

Electricity is charged for by the **kilowatt-hour.** It is measured by an electric meter which records the number of kilowatt-hours of electric current used. The electric meter has four dials. In reading a meter, begin at the left. On each dial, read the number to which the pointer points. If the pointer is between two numbers, read the smaller number.



1. The meter at the left reads 6421 kw-hr. Read the meter at the right.

How to Find the Cost of Electricity

An electric meter read 8643 kw-hr on Nov. 30 and 8750 kw-hr on Dec. 31. Find the charge for December at $6\frac{1}{2}¢$ per kw-hr for the first 40 kw-hr, $5\frac{1}{2}¢$ per kw-hr for the next 40 kw-hr, and $4¢$ for each kilowatt-hour over 80.

8750 kw-hr — 8643 kw-hr = 107 kw-hr Find the difference between the two readings.

\$.065 × 40 =	\$2.60	Multiply each rate by the number of kilowatt-hours at that rate.
\$.055 × 40 =	2.20	
\$.04 × 27 =	1.08	
	\$5.88	Add the separate charges.
	charge for December	

2. An electric meter reads 6422 kw-hr one month and 6568 kw-hr the next. Find the charge for electricity at $5\frac{1}{2}¢$ per kw-hr for the first 15 kw-hr, $4¢$ for each of the next 45 kw-hr, and $2\frac{1}{2}¢$ per kw-hr for the next 140 kw-hr.

Gas used in the home is charged for by the **cubic foot (cu.ft.)** and is measured by a gas meter which records the number of cubic feet of gas used. A gas meter has three dials. The dial at the right measures by the hundred cubic feet. Once around the dial equals 1000 cu. ft. The center dial measures by the thousand cubic feet. The dial at the left measures by the ten-thousand cubic feet. In reading a gas meter, begin at the left. If the pointer is between two numbers, read the smaller number.

1. The meter at the left reads 43,400 cu. ft. Explain. Then read the meter at the right.

How to Find the Cost of Gas

Last month, a gas meter read 8200 cu. ft. This month, it reads 9800 cu. ft. Find the charge at \$1.35 for the first 1000 cu. ft., and \$1.10 per M for over 1000 cu. ft.

9800 cu. ft. — 8200 cu. ft. = 1600 cu. ft., or 1.6 M cu. ft.	Find the difference between the two readings.
\$1.35 × 1 = \$1.35	Multiply each rate by the number of thousand cubic feet at that rate.
\$1.10 × .6 = .66	
\$2.01 charge for gas	Add the separate charges.

2. Find the cost of 2800 cu. ft. of gas at 50¢ for the first 286 cu. ft. and \$1.70 per 1000 cu. ft. for the balance. Find the cost of 1700 cu. ft. at these rates.

Water is charged for by the cubic foot or by the gallon.

3. Find the cost of: (a) 1500 cu. ft. of water at \$1.50 for the first 1250 cu. ft. and 12¢ per 100 cu. ft. for the balance; (b) 11,250 gal. of water at \$.24 per 1000 gal.

This washing machine is for sale on the installment, or easy payment, plan.

MRS. TRACY: "I wish I could have that washing machine. It would be a wonderful help. The sign says $57 cash, or a down payment of $10 and $10 a month for 5 months."

DEALER: "That's right. Some people really need things but do not have the money on hand to pay for them. Buying on the **installment,** or **easy payment, plan** allows them to get what they need when they need it."

MRS. TRACY: "That sounds fine, but suppose a person can't make the payments when they are due? What happens then?"

DEALER: "We would take the washing machine. When a person buys on the installment plan, he signs a contract agreeing that if he is unable to make the payments when due the dealer may take back the article and keep the money that has been paid."

MRS. TRACY: "Well, now let's see. Five payments of $10 each is $50. With the down payment of $10, I would be paying $60 for the machine, or $3 more than the cash price."

DEALER: "Yes. You see, the machine is worth $57 to us. You take it, and pay us only $10 down. So we are really lending you $47, for which we charge you $3, which is equal to interest of about 6% a year on the $47."

MRS. TRACY: "That means that I would be paying a year's interest on the full amount of the loan, even though I was reducing this amount by monthly payments all of which would be completed in 5 months."

DEALER: "That is true. But the $3, which we call a **carrying charge,** pays not only the interest but also the cost of looking up your credit to see if you pay your bills promptly and of carrying your account on our books. It pays, too, for the risk of having to take back the used machine."

Before buying on the installment plan, a person should figure what **rate** of carrying charge he will be paying. If the rate is too high, it may be wiser for him to borrow the money at a bank at the usual interest rates.

How to Find the Yearly Rate of Carrying Charge in Installment Buying

The cash price of a washing machine is $57. The installment price is $10 down and $10 a month for 5 mo., or $60. What is the yearly rate of carrying charge?

$60 − $57 = $3 carrying charge	Find the carrying charge.
$60 − $10 = $50 owed 1 mo. $50 − $10 = $40 owed 1 mo. $40 − $10 = $30 owed 1 mo. $30 − $10 = $20 owed 1 mo. $20 − $10 = $10 owed 1 mo. $150 total amount owed for 1 mo.	Find the amount owed each month. Find the total of these amounts. Owing each of these amounts for 1 mo. is the same as owing the total of these amounts for 1 mo., and the carrying charge may be considered as the interest on this total for 1 mo.
$3 ÷ $150 = 2% monthly rate of carrying charge	
2% × 12 = 24% yearly rate of carrying charge	

Find the monthly rate of carrying charge by dividing the carrying charge by the total amount owed for 1 mo.

To find the yearly rate of carrying charge, multiply the monthly rate by 12.

*Optional material.

1. A friend of Mrs. Tracy's offered to lend her the $57 needed to pay cash for the washer, at 6% interest for 5 mo. How much interest would Mrs. Tracy pay? About how many times greater was the installment rate than the interest rate?

2. Mr. Crandall buys a new car, giving his old car as cash payment, and agreeing to pay the balance of $653.94 at the rate of $36.33 a month. How many months will it take to complete the payments?

3. Mrs. Kane bought a kitchen cabinet on the installment plan for $45. The cash price was $39.50. What per cent of increase did she pay over the cash price?

4. Charles can buy a bicycle for $29.50 cash, or for a down payment of $6 and $4.50 a month for 6 mo. How much more is the installment price than the cash price?

*5. A radio is priced at $45 cash, or at $48 if purchased on the easy payment plan of $13 down and the balance in 5 equal monthly installments. How much is each monthly installment? What is the yearly rate of carrying charge?

QUESTIONS ABOUT INSTALLMENT BUYING

Before buying on the installment plan, give careful thought to these questions:

1. Do I need the article now, or can I wait until I have saved enough money to pay cash for it?

2. Will my budget stand the expense of installment payments?

3. Can I meet each payment when due?

4. Will I be paying too high a rate of carrying charge?

*Optional material.

Doris Henson was with her father when he drove out to see some new houses in a suburb called Westland. "What does that sign at the right mean?" asked Doris.

"It means," said Mr. Henson, "that we can buy one of these homes on the installment plan."

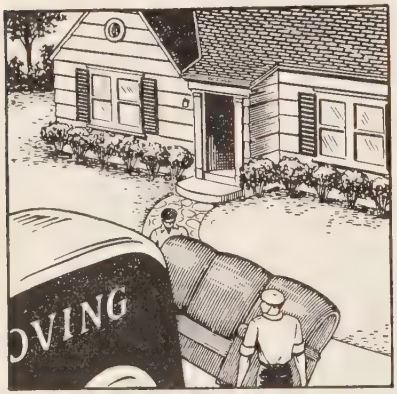
The real-estate agent showed Mr. Henson and Doris through one home and explained the terms of payment. The buyer was required to pay $2500 in cash and to give a mortgage for the balance of $5000. This balance he was to pay in monthly installments of $40 each, plus 6% interest on the unpaid portion of the principal.

At the end of the first month after purchase, the buyer would pay $40 plus $25, interest on $5000 at 6% for 1 mo., or a total payment of $65. The balance then due on the principal would be $5000 − $40, or $4960. At the end of the second month, the buyer would pay $40 + $24.80, interest on $4960 at 6% for 1 mo., or a total of $64.80. As the principal would decrease $40 each month, the interest, too, would decrease; and each total monthly payment would be smaller than the one before it.

1. Check the third payment in the table below, and continue the table to cover payments for the first year.

End of	Prin.	Payment on Prin.	Int. on Prin.	Total Monthly Payment
st mo.	$5000	$40	$25	$65
nd mo.	$4960	$40	$24.80	$64.80
rd mo.	$4920	$40	$24.60	$64.60

2. How many months would it take to pay off the mortgage on the terms given above? How many years?



The Hensons move into their new home.

Mr. Henson finally bought a new home in the Riverside section. Here, too, part of the purchase price was to be paid in monthly installments, but the plan of payment was different.

The price of Mr. Henson's new home was \$8400, with a down payment of \$2800. Mr. Henson was required to give a mortgage for the balance of \$5600 and to make monthly payments of \$84, including interest at 6% on unpaid balances, until the mortgage was paid.

The interest on \$5600 at 6% for 1 mo. is \$28. Of the first monthly payment of \$84 which Mr. Henson made, \$28 was interest, and the rest, or \$56, was credited against the principal. \$5600 − \$56 = \$5544, new principal. Of the second monthly payment, \$27.72 was interest. Why? \$84 − \$27.72 = \$56.28 applied on the principal. \$5544 − \$56.28 = \$5487.72, new principal.

Notice that the interest decreases each month, but since the total monthly payment remains unchanged the principal is reduced by a larger amount each month.

1. Check the third month's payment in the table below; then continue the table to cover payments for 6 months.

End of	Prin.	Payment on Prin.	Int. on Prin.	Total Monthly Payment
1st mo.	\$5600	\$56	\$28	\$84
2nd mo.	\$5544	\$56.28	\$27.72	\$84
3rd mo.	\$5487.72	\$56.56	\$27.44	\$84

People who need money to build homes often borrow from **building and loan associations.** A person who borrows from such an association must give a mortgage as security for the loan and must agree to pay interest monthly on the full amount of the loan until the debt is canceled.

To pay this loan, the borrower must buy shares in the association equal in face value to the amount of the loan. He pays for these shares in monthly installments of $.50 for each $100 share and $1 for each $200 share. These payments earn compound interest, which is allowed to accumulate toward payment for the shares. The shares thus become paid up more quickly than if no interest were earned. When the shares are paid up, they are turned over to the association to cancel the loan.

A building and loan association lent Mr. Finley $4000 to help him build a house. He gave the association a mortgage for $4000 on which he agreed to pay interest monthly at the rate of 6% a year. He also agreed to buy shares in the association equal in face value to $4000, the amount of his loan. The face value of each share was $200, so Mr. Finley had to buy 4000 ÷ 200, or 20 shares. The monthly payment on these shares was $1 for each share. Mr. Finley's total monthly payment was figured as follows:

Interest on $4000 for 1 mo. at 6% = $20
Payment on 20 shares at $1 each = 20
Total monthly payment = $40

When Mr. Finley's shares were finally paid for, he turned them over to the association to cancel his mortgage.

Because of the compound interest which building and loan shares earn, people also buy them simply as an investment. The interest decreases the cost of the shares. When paid up, the shares may be turned in for their face value or left with the association to earn further interest.

1. Mr. Martin bought as an investment twenty \$100 shares in a building and loan association. He paid 50¢ a month for each share. (a) How much did he pay on his shares in 1 yr.? In $12\frac{1}{2}$ yr.? (b) If the shares became paid up in $12\frac{1}{2}$ yr., how much interest did he receive on his investment?

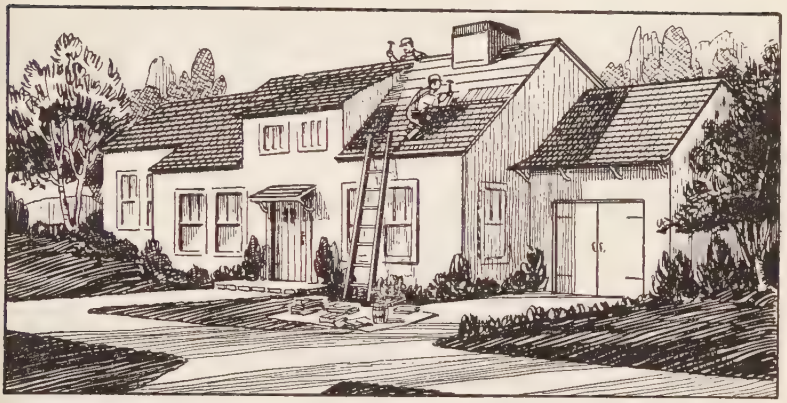
2. Mr. Curtis borrowed \$3600 from a building and loan association, purchasing \$200 shares to cover the loan. (a) How many \$200 shares did he buy in the association? (b) If he made monthly payments of \$1 per share and interest at 6% on the loan, how much was each total monthly payment? (c) If the shares became paid up in 169 mo., how much did Mr. Curtis pay for them? (d) How much in all did he pay for the loan?

Organizations similar to building and loan associations are **federal savings and loan associations.** These are under the supervision of the federal government. They offer an easy and economical method of paying for homes as well as a convenient and safe way for people with small savings to invest money. Instead of buying shares in the organization, the borrower repays his loan in monthly installments under a plan like the one described on page 500.

People who invest in federal savings and loan associations do so by depositing money the same as in a savings account, and receive compound interest on their savings.

3. If you wished to borrow money to build a house, which plan of repayment would you prefer, the federal savings and loan installment plan or a building and loan plan? Why? Assume the same rate of interest in each case.

4. Find out what types of building and loan associations are most common in your state. If there is a federal savings and loan association near you, report to the class on the plans it offers for saving and for loans.



When Mr. Sheridan was making out his budget for the new year, he allowed $30 a month for shelter. "Why?" asked Walter. "We own our home."

Mr. Sheridan explained that the expenses on their home, including the cost of **upkeep,** or repairs, averaged about $40 a month during the preceding year, but that he thought $30 a month would cover all expenses for the present year.

1. Mr. Sheridan had Walter work out the following statement of expense on their home for the preceding year. Copy the statement, fill in the missing amounts, and find the yearly and the average monthly expense.

Insurance. $7000 at 65¢ per $100	$_____
Taxes. Assessed valuation, $6400; tax rate, $34.15 per $1000	_____
Repairing sidewalk. 2 da. work at $3.25 a day; materials $10.75	_____
New roof for house and garage	135.00
Minor repairs. $5\frac{1}{2}$ da. work at $4.50; materials, $16.35	_____
Miscellaneous	11.89

2. Make a list of common upkeep expenses on a home.

Try to work these problems without using pencil and paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. Mrs. Delano bought a dining room suite for $240, paying $40 cash and the balance in 10 equal installments. How much was each installment?

2. Last year, the Morrisons spent an average of $25 a month for clothing. Their monthly income was $200. What per cent of their income was spent for clothing?

3. A sales slip showed these items: 6 handkerchiefs at 15¢ each; 2 yd. of ribbon at 20¢ a yard; 1 pincushion, 10¢. What was the amount of the sales slip?

4. Ellen earns $18 a week. She allows $8 for room and board; $4 for carfare, lunches, and entertainment; $3 for clothes; and the rest for savings. How much does she save? What per cent of her salary does she save?

5. Mrs. Judson's household account showed $85 cash on hand Mar. 1; receipts for March, $170; expenses for the month, $150. How much was the cash on hand Apr. 1?

6. An electric refrigerator costs $135 cash, or $7.50 per month for 20 months. How much is saved by paying cash?

7. Mr. Elkins' expenses on his home last year were: taxes, $80; insurance, $30; repairs, $130. What monthly rent did these expenses equal?

8. The Carsons' gas meter read 36,500 cu. ft. one month and 39,500 cu. ft. the next month. The charge for gas was $1.25 per 1000 cu. ft. What was the amount of their gas bill?

Work the examples in column A. If you fail on any example, do the example opposite in column B. If you still make mistakes, review the pages in column C.

A Test	B Drill	C Review
1. Finding the Cost of Electricity		
Meter readings: One month, 3428 Next month, 3492 Charge: $6\frac{1}{2}¢$ per kw-hr for the first 40 kw-hr; balance, $4\frac{1}{2}¢$ per kw-hr	Meter readings: One month, 4504 Next month, 4582 Charge: $5\frac{1}{2}¢$ per kw-hr for the first 40 kw-hr; balance, $4\frac{1}{2}¢$ per kw-hr	494
2. Finding the Cost of Gas		
Meter readings: July 1, 15,840 cu. ft. Aug. 1, 17,620 cu. ft. Charge: \$1.26 per 1000 cu. ft.	Meter readings: Apr. 15, 22,090 cu. ft. May 15, 24,980 cu. ft. Charge: \$1.40 per 1000 cu. ft.	495
3. Buying on the Installment Plan		
Cash price, \$160 Easy payment price: \$7.50 down, balance in 12 monthly payments of \$13.50 each Carrying charge, ?	Cash price, \$310 Easy payment price: \$35 down, balance in 10 monthly payments of \$29.50 each Carrying charge, ?	496
4. Borrowing from a Building and Loan Association		
Amount borrowed, \$3000 Shares purchased, 15 \$200 shares at \$1 per share per month Interest payable monthly at 6% Total monthly payment, ?	Amount borrowed, \$4000 Shares purchased, 20 \$200 shares at \$1 per share per month Interest payable monthly at 6% Total monthly payment, ?	501

See if you can do each of these tests in the time given.

SET 1. ADDITION (4 minutes without copying)

1.	2.	3.	4.
15983	57064	917.6	697.2
2064	39339	12.05	308.8
87944	4589	840.37	970.58
62396	62565	2.7	39.362
29989	5840	563.19	336.5
4857	71646	7.39	.878

SET 2. SUBTRACTION (5 minutes without copying)

1.	2.	3.	4.
345563	874617	793971	981986
147918	709475	216492	803897

5.	6.	7.	8.
755.413	126.695	6300.07	145.
281.54	40.627	5719.23	95.201

SET 3. MULTIPLICATION (12 minutes after copying)

1.	2.	3.	4.
2674	4289	7468	15039
295	460	798	187

5.	6.	7.	8.
12367	391.85	90.135	7309.8
.063	5.06	39.2	8.94

SET 4. DIVISION (25 minutes after copying)

1. $17\overline{)18666}$	2. $88\overline{)40150}$	3. $54\overline{)35053}$
4. $315\overline{)23814}$	5. $564\overline{)491244}$	6. $672\overline{)496272}$
7. $.92\overline{)733.7}$	8. $38.6\overline{)3811.83}$	9. $.758\overline{)659119}$

1. How much change should I receive from a dollar given in payment for $2\frac{1}{2}$ lb. of beefsteak at 30¢ a pound?

Tell how you would work this problem. Then work it without using pencil and paper.

2. A building valued at \$16,000 was insured for $\frac{3}{4}$ of its value. The three-year fire insurance rate was 90¢ per \$100 of insured value. What was the cost of the insurance for three years?

Before working, name the steps in this problem.

3. A pear grower sent a shipment of pears to a commission agent in New York. The agent sold the fruit, deducted freight and cartage and his commission, and remitted the proceeds. Find the proceeds.

Supply the missing values, and then work the problem.

4. The property in a village is assessed at \$1,200,000. The amount of school tax to be raised is \$9000. Mr. Wilcox's property in the village is assessed at \$15,000.

Write two questions for the above statements to make a problem. Solve the problem you make.

5. In 7 tests, Fred received 71%, 92%, 100%, 64%, 88%, and 70%. Find his average mark for the tests.

Correct the mistake in this problem. Work the problem, and draw a line graph comparing Fred's marks.

6. A high-school boy sells magazines at 25¢ a copy. He sold 40 copies one week and was allowed 6¢ a copy commission. What was his rate of commission?

Tell what number in this problem is not needed to find the answer. Then solve the problem.

1. On Mar. 1, you deposit \$5.35 in a bank in your town. Make out your deposit slip.

2. On Mar. 10, you give the Black Shoe Store your check for \$4 in payment for shoes. (a) Write the check, omitting your signature. (b) Write your receipt from the shoe store. (c) Write the Black Shoe Store's endorsement of the check for deposit in their bank.

Find the answers, using short methods:

3. 25% of \$884 = ?

4. $1000 \times 65.8 = ?$

5. $16 = 33\frac{1}{3}\%$ of ?

6. $92.8 \div 10 = ?$

Estimate the answer to each of the following four examples, and tell which answer in parentheses is nearest correct. Work the example to prove that you are right.

7. $24\frac{1}{2}\%$ of \$1256 = (\$30.76; \$307.72; \$320.40)

8. $59.6 \times 2.03 =$ (120.988; 1209.88; 12.0988)

9. $25896 \div 26 =$ (826; 646; 996)

10. $\frac{2}{2}\frac{1}{5}$ of 2550 = (2072; 2142; 204)

11. If 1 doz. oranges cost 38¢, what will 8 oranges cost?

12. What is $\frac{1}{4}\%$ of \$250?

13. What is $187\frac{1}{2}\%$ of 160?

14. Find the interest on \$624.50 at 6% for 84 da.

15. What is the difference between finding a trade discount of 6% of \$400 and finding the bank discount on \$400 at 6%? Make and work two problems illustrating this difference.

16. Property which cost \$8000 is sold for \$10,000. The expense of sale is 4% of the selling price. What is the profit? Find the per cent of profit based (a) on the cost; (b) on the selling price.

1. A man paid \$39.38 interest for 1 yr. on a loan of \$716. What yearly rate of interest did he pay?

2. A concrete road costs \$135 a rod. At this rate, what will be the cost of a concrete road $2\frac{1}{2}$ mi. long?

3. At what price must a dealer sell a mirror for which he paid \$4.50, if he allows 20% for overhead and 20% for profit, based on the selling price?

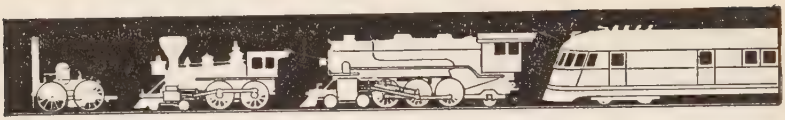
4. On Dec. 15, Mr. Charles Williams borrowed \$1500 from the First National Bank. He gave the bank a promissory note, due in 90 days, with interest at 5%. Write the note. Find the amount that will be due at maturity.

5. Mr. Graves engaged a real estate agent to buy 3 pieces of property for him. For the first, the agent paid \$7200; for the second, \$6500; and for the third, \$5500. The agent's commission was 4%. Find the commission, and the total cost to Mr. Graves.

6. A school purchased 8 desks at \$36 each, 10 chairs at \$12.75 each, and 3 tables at \$17.50 each, and was allowed a trade discount of $33\frac{1}{3}\%$, and 2% for prompt payment. Find the net amount the school paid.

7. Find the bank discount and the proceeds of a note for \$2000, dated May 21, for 90 da., with interest at 5%, if discounted by a bank on June 10 at 6%.

8. What is the difference between the amount of \$1200 for $2\frac{1}{2}$ yr. at $2\frac{1}{2}\%$ compounded semiannually, and at 4% simple interest? What is the difference in the interest earned?



1. Find the sum of $12\frac{1}{2}$, $3\frac{2}{3}$, $7\frac{1}{2}$, and $\frac{5}{6}$.

2. Subtract 790 bu. 3 pk. from 5000 bu.

3. Express $\frac{3}{7}$ as a per cent.

4. Divide 2.0835 by 4.5.

5. $4\frac{1}{6} \times 2\frac{1}{2} \times 3\frac{3}{5} = ?$

6. $16.5 = \frac{3}{5}\%$ of ?

7. Find the cost of 3100 cu. ft. of gas at \$1.32 for the first 1000 cu. ft. and \$.95 per M for the balance.

8. What are the amounts allowed in a \$250 monthly budget as follows: Food, 20%; clothes, 15%; rent, 20%; operating expenses, 12%; other expenses, 18%; balance, savings?

9. Find the cost of fire insurance for 1 year on a \$7500 house insured for 80% of its value at \$.35 per \$100.

10. Find the tax on property valued at \$9600 and assessed at $\frac{2}{3}$ of its value if the rate is $17\frac{1}{2}$ mills per \$1.

11. An electric meter read 5189 one month and 5228 the next. Find the cost of the current used at $5\frac{1}{2}$¢ per kw-hr.

12. Mr. Crandall can buy a tractor for \$520 cash or for a down payment of \$120 and 6 monthly payments of \$72.50 each. How much will he save by paying the cash price?

13. Last year, the expenses on our home were: taxes, \$32.50 per M on \$8000; \$120 interest on a mortgage; \$152 for repairs. What was the average monthly expense?

14. What is the cost of 120 shares of stock at $63\frac{1}{8}$, brokerage 19¢ per share?

15. Find the interest on \$716 for 93 da. at 5%.

*Record your score on your Progress Chart. See page 321.

UNIT VIII. GEOMETRIC FIGURES AND MEASUREMENTS

One way in which geometric figures are used.

Wherever you look, you see that all objects have shape, or form. This page is shaped like a rectangle. Pennants are often shaped like triangles or rectangles. Faces of watches are usually shaped like circles, rectangles, or squares.

In science, in industry, and in construction work, there is often a close connection between the shape of an object and its usefulness. Parts of machinery which are shaped like rectangles usually either are stationary or move back and forth; parts which are circular usually rotate or turn. Parts of the frames of bridges are often shaped like triangles because this form is strong and rigid.

Circles, rectangles, triangles, and other *plane*, or flat, figures, and all *solid* figures, are called *geometric figures*. The study of geometric figures and their measurements is called *geometry*.

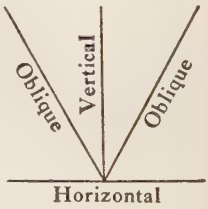
Geometry is used in drawing plans, in measuring surfaces, in building, and in many other ways.

Straight Line | Broken Line | Curved Lines | Parallel Lines

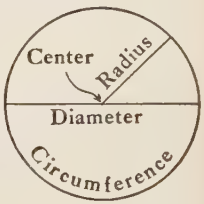
Give the name of each of the following:

1. A line which does not change its direction.
2. A line formed by several connected straight lines.
3. A line that constantly changes its direction.
4. Lines that never meet, however far they are extended.

Complete the following:



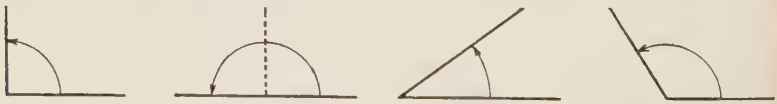
5. A straight right and left line is a ___.
6. A straight up and down line is a ___.
7. A straight line that is neither ___ nor ___ is an ___.



A Circle

8. A figure bounded by a curved line every point of which is the same distance from the center is a ___. The line which bounds it is the ___. The distance from the center to any point on the circumference is the ___. The distance across a circle through the center is the ___.

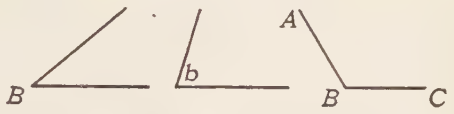
When two straight lines meet, they form an *angle*. The angle is the opening between the lines which form its sides. The point where the two lines meet is called the *vertex* of the angle.



Right Angle | Straight Angle | Acute Angle | Obtuse Angle

9. What kind of angle forms a square corner?
10. The sides of what kind of angle form a straight line?
11. What kind of angle is less than a right angle?
12. What kind of angle is greater than a right angle but less than a straight angle?

1. This diagram shows the three ways of naming angles. How, and at what point or points, is each of the three angles lettered?



When three letters are used to name an angle, the letter at the vertex is read in the middle, as *angle ABC* or *angle CBA*.

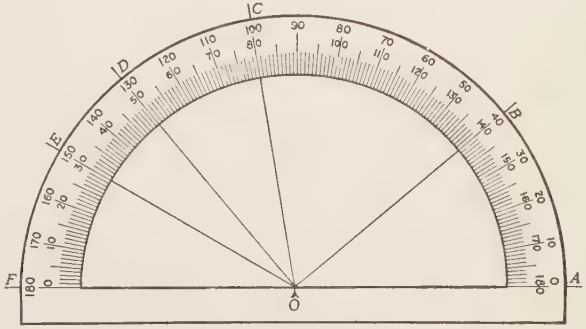
The sign for angle is ∠ ; for angles, ∠s. Angles are measured in *degrees* (°). Every right angle contains 90°.

2. How many degrees are there in a straight angle? In all the angles that can be drawn around a point?

90°	90°
90°	90°

A **protractor** is commonly used to measure angles. It is numbered in degrees from right to left and left to right.

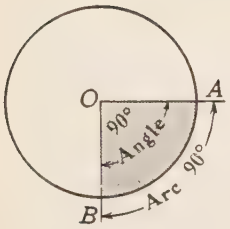
3. Tell how the protractor is used to measure ∠ *AOD* at the right. How many degrees in angle *AOD*? How many in ∠ *AOB*? ∠ *DOB*? ∠ *COF*?



Measuring Angles with a Protractor.

4. Draw a line and label it *AB*. Using a protractor and a ruler, draw lines from *A* making these angles with *AB*: 65°, 87°, 13°, 45°, 117°, 51°, 168°, 145°, 175°, 8°.

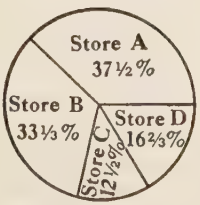
5. Using a ruler only, draw an angle with sides 2″ long. Measure the angle with a protractor. Then extend the lengths of the sides, and measure the angle again. Do the lengths of the sides of an angle affect the size of the angle?



There are 360° in the circumference of any circle and 360° in all the angles that can be drawn at its center. When two radii are drawn within a circle, the **arc,** or part of the circumference cut off, has the same number of degrees as the angle formed by the radii. In the circle at the left, $\angle BOA$ and arc BA each contain 90°. The shaded **sector** of the circle is $\frac{1}{4}$ the area, or surface, of the circle, and its arc is $\frac{1}{4}$ of the circumference. These facts help in making **circle graphs.**

How to Make a Circle Graph

In one month, a company's four stores had sales of \$96,000. Store A's sales were \$36,000; Store B's, \$32,000; Store C's, \$12,000; Store D's, \$16,000. Make a circle graph showing what per cent each store's sales were of the total sales.



Sales of Four Stores

Set the points of your compasses to any desired radius. Draw a circle, and mark the point at the center.

Find what fraction or per cent each part is of the whole.

Multiply 360° by each fraction or per cent to find the number of degrees needed to represent the part.

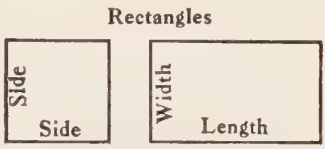
Using a protractor, draw at the center of the circle an angle containing the number of degrees needed to represent the first part. In the same way, draw angles to represent the other parts.

Label each section of the graph. Label the graph.

1. Make a circle graph budgeting \$24 as follows: Rent, \$6; Food, \$8; Clothing, \$3; Pleasure, \$1; Savings, \$6.

1. What is a figure with four straight sides and four right angles?

2. How does a square differ from an oblong?



Any figure having four straight sides with opposite sides parallel and equal is a *parallelogram.*

3. Is a rectangle a parallelogram?

4. What name is given to the side on which a parallelogram rests? What is the **perpendicular** distance, or the shortest distance, between the base and the opposite side called?

Study the above figures; then complete the following:

5. A figure with six straight sides is a ____.

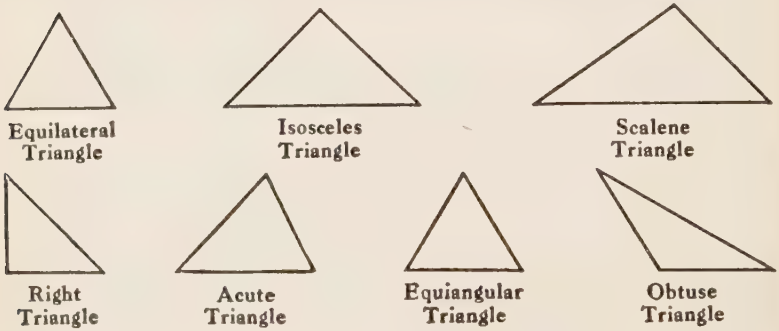
6. A figure with eight straight sides is an ____.

7. A four-sided figure with two parallel sides and two sides that are not parallel is a ____. The two parallel sides are called the ____. The perpendicular distance between the parallel sides is the ____.

8. A figure with three straight sides is a ____. The side on which it rests is the ____. The perpendicular distance between the base of a triangle and its highest point is the ____.

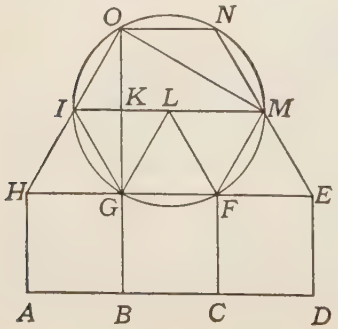
Triangles are named according to (a) the number of sides that are equal, or (b) the size of their angles. The sign for triangle is $\triangle$; for triangles, $\triangleq$.

Study these triangles; then complete the statements below:



9. A triangle with all three sides equal is an ___ triangle.
10. A triangle with two sides equal is an ___ triangle.
11. A triangle with no two sides equal is a ___ triangle.
12. A triangle having one right angle is a ___ triangle.
13. A triangle having three acute angles is an ___ triangle.
14. A triangle having three equal angles is an ___ triangle.
15. A triangle having one obtuse angle is an ___ triangle.

In the drawing below, name the following by letters; as, a square *ABGH*.



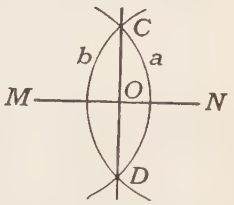
16. A square
17. A hexagon
18. An oblong
19. A circle
20. A trapezoid
21. A rectangle
22. A parallelogram
23. A right triangle
24. An obtuse triangle
25. An equilateral triangle
26. An equiangular triangle
27. A scalene triangle
28. An isosceles triangle

In drawing plans and in making designs, it is often necessary to **construct** lines, angles, and other geometric figures.

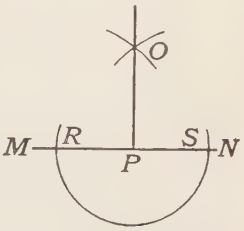
In the following constructions, a ruler is used to draw straight lines, and the compasses to draw arcs and circles. Read each explanation carefully, study the drawing, and then do the construction on separate paper. Make your drawings larger than those shown.

Hereafter, the word **line** is used to mean **straight line.**

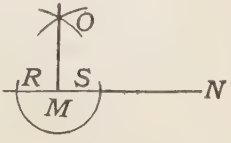
When we divide a line or an angle into two equal parts, we **bisect** it.



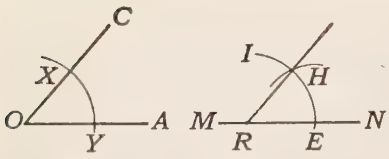
To bisect any line, such as MN: Draw arc a with M as center, using a radius greater than half of MN. Using the same radius, draw arc b with N as center. Draw line CD connecting the points where the arcs **intersect,** or cross. By measuring, prove that line CD bisects line MN at O. The bisector CD is a **perpendicular** because it makes right angles with MN.



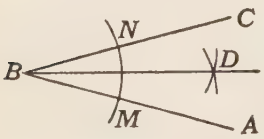
To construct a line perpendicular to another line at a given point: Draw a line MN, and mark a given point P. With P as center and any convenient radius, draw an arc cutting MN at R and S. With R as center and a radius greater than RP, draw an arc above MN. Using S as center and the same radius, draw an arc cutting the first arc at O. Draw a line connecting O and P. Line OP is perpendicular to line MN at P.



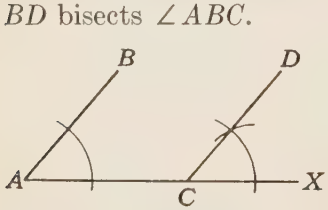
To draw a perpendicular at the end of a line, first extend the line as shown. Then proceed as described above.



To construct an angle equal to a given angle, such as $\angle AOC$: Draw a line, MN. With O as center and any convenient radius, draw an arc cutting OC and OA at X and Y. With the same radius and using any point in MN, such as R, as center, draw arc IE cutting MN at E. With the compasses, measure the distance between X and Y in the given angle. With this distance as radius and E as center, draw an arc cutting IE at H. Draw a line from R through H. $\angle ERH = \angle AOC$.



To bisect a given angle, such as $\angle ABC$: With vertex B as center and any convenient radius, draw an arc cutting AB and BC at M and N. With the same radius and M and N as centers, draw two arcs intersecting at D. Draw line BD. BD bisects $\angle ABC$.



To draw a line parallel to another line, such as AB, using the compasses: Draw line AX making any desired angle with AB. At any point on AX, such as C, draw $\angle XCD$ equal to $\angle CAB$. Line CD is parallel to line AB.

1. Divide a line 5″ long into 4 equal parts, using the compasses. Check the length of each part with your ruler.

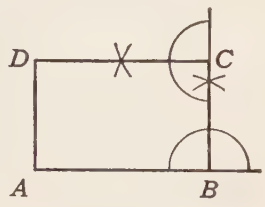
2. Draw an oblique line 2″ long, and draw a perpendicular at each end of the line.

3. Using a protractor, draw a 97° angle. Using the compasses, draw an angle equal to this angle, and bisect it.

4. Draw a line. Using compasses and ruler, draw parallel lines 1″ either side of the first line.

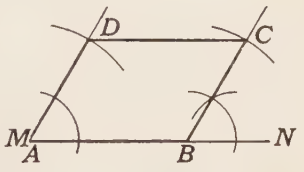
Read each explanation carefully. Then do the exercise.

To construct a rectangle of any given size: Draw line AB equal to the length of the rectangle. Erect a perpendicular at B, and mark off BC equal to the width of the rectangle. At C, erect a perpendicular to BC, and mark off CD equal to AB. Connect D and A by a line. $ABCD$ is the required rectangle.



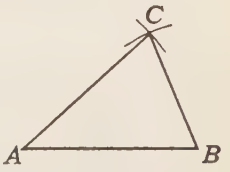
1. Draw a rectangle $1\frac{1}{2}''$ long and $\frac{3}{4}''$ wide.

To construct a parallelogram having sides of given lengths: Draw a line MN. From any point on MN, such as A, mark off AB equal to the base of the parallelogram. Draw lines at A and B forming equal angles with MN. On the lines just drawn, mark off AD and BC equal to the sides of the parallelogram. Connect C and D by a line. $ABCD$ is the required parallelogram.



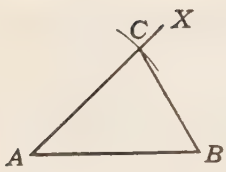
2. Draw a parallelogram with base $3''$ and slant sides $1\frac{1}{2}''$.

To construct a triangle, given the lengths of its sides: Draw a line AB equal to the first side. With A as center and a radius equal to the second side, draw an arc. With B as center and a radius equal to the third side, draw an arc cutting the first arc at C. Draw lines AC and BC. $\triangle ABC$ is the required triangle.



3. Draw a triangle having sides $2''$, $3''$, and $4''$.

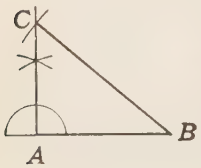
In any triangle, the sum of the lengths of any two of the sides is always greater than the length of the third side. Could you draw a triangle $3'' \times 2'' \times 1''$? Try it.



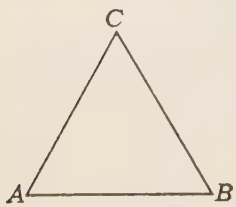
To construct a triangle when given the lengths of two of its sides and the size of the angle included between them: Draw line AB equal to one of the given sides. Using a protractor, draw at A $\angle BAX$ equal to the given angle. On AX, mark off AC equal to the other given side. Connect B and C by a line. $\triangle ABC$ is the required triangle.

4. Draw a triangle having sides 1″ and $1\frac{1}{2}$″ forming an angle of 72°.

5. Draw a right triangle having sides 2″ and $1\frac{1}{4}$″ forming a right angle.



6. Study the drawing at the left. Then explain how to draw a right triangle, given the lengths of the base (AB) and the side opposite the right angle (BC).

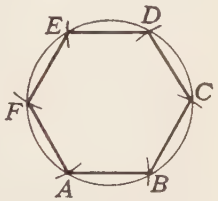


To construct a triangle when given the sizes of two of its angles and the length of the side included between them: Draw line AB equal to the given side. Using a protractor, draw at A an angle equal to one of the given angles. At B, draw an angle equal to the other given angle. Extend the sides of the angles until they meet at C. $\triangle ABC$ is the required triangle.

7. Draw a triangle having a side $1\frac{1}{2}$″ long included between an angle of 58° and an angle of 47°.

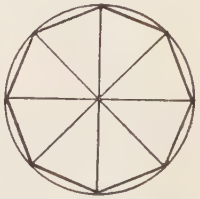
8. Draw a triangle $2'' \times 2\frac{1}{4}'' \times 1\frac{3}{4}''$. Using each side as a base, draw the three heights of the triangle. The lines should meet in a point. Draw another triangle of the same size, and bisect the angles. Do the bisectors meet in a point?

A hexagon having six equal sides and six equal angles is a **regular hexagon.** When a hexagon is drawn within a circle and every corner of the hexagon touches the circle, the hexagon is said to be **inscribed** in the circle.



To construct a regular hexagon having sides of a given length: Draw a circle having a radius equal to a side of the hexagon. Using any point A of the circumference as center and the same radius as that of the circle, draw an arc cutting the circumference at B. Using B as center and the same radius, draw another arc cutting the circumference at C. In the same way, locate points D, E, and F. The arc drawn at F should cut the circle at A. Connect each two successive points by a line. Figure $ABCDEF$ is the required regular hexagon.

9. Draw a regular hexagon having sides $1\frac{1}{2}''$ long.



To inscribe an octagon in a circle of a given radius: Draw the circle. Draw the horizontal diameter of the circle. Then draw another diameter, perpendicular to the first. Draw two more diameters by bisecting each right angle thus formed, extending the bisectors to the circumference. Draw lines connecting the points at which the four diameters meet the circumference.

10. Inscribe an octagon in a circle of $\frac{7}{8}''$ radius.

Construct the following:

11. A rectangle $1\frac{3}{4}'' \times \frac{7}{8}''$. **12.** A triangle $1\frac{1}{2}'' \times 2'' \times 2\frac{1}{4}''$.

13. A parallelogram with base $2''$ and slant sides $1\frac{1}{2}''$.

14. A triangle with sides $1\frac{3}{4}''$ and $1\frac{1}{4}''$ forming a $57°$ angle.

15. A regular hexagon with sides $1\frac{1}{4}''$ long.

16. An octagon in a circle having a $1\frac{1}{2}''$ diameter.

1. Look around you in the classroom and point out examples of different kinds of lines, angles, and plane geometric figures.

2. Bring to class pictures which illustrate the use of geometric figures in building, art, home furnishings, and industry. Suggest the reasons why certain objects are made in certain shapes.

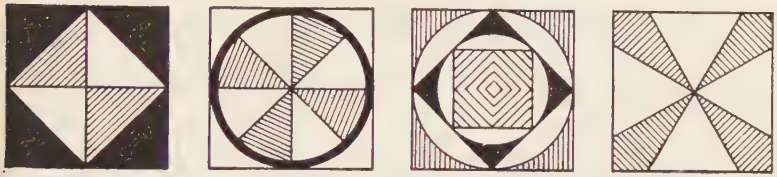
3. Since there are 360° in the circumference of any circle, what fractional part of the circumference is 1°? Is the length of 1° in the circumference of a small circle the same length as 1° in a large circle? Explain.

4. The distance around the earth at the Equator is about 25,000 miles. About what distance in miles is 1° at the Equator? Is 1° of east-west distance the same length on all parts of the earth's surface? Explain.

5. Draw a line 2″ long, and place a point 1″ above the line. Using a protractor, draw a perpendicular from the line to the point. A perpendicular is the shortest distance from a point to a line.

When drawing lines, angles, and straight-line figures, architects and others often use the **T-square** and the **draftsman's triangle.** Carpenters use a **carpenter's square** to draw accurate cutting lines on boards.

6. Using T-square or ruler and draftsman's triangle, draw: (a) a rectangle $2\frac{1}{2}'' \times 1\frac{3}{4}''$; (b) a 2″ square; (c) a parallelogram having a base 2″ long and slant sides 1″ long.



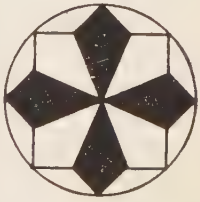
1. What geometric figures are used in the above designs? Tell how each design was drawn, and make similar ones.

2. Using circles and squares, draw two designs different from those above.

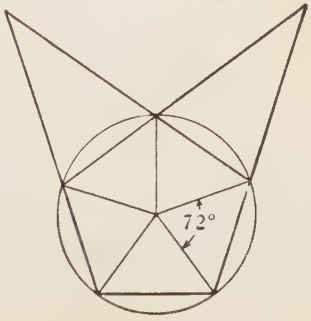
3. Study the first design at the right, which shows a 6-point star drawn within a hexagon. Draw the design, using different shading.

4. Make an original design, using a circle, a hexagon, and a triangle.

5. What geometric figures do you see in the next design? Draw an octagon in a circle, and make a similar design. Shade your design differently.



6. Draw a circle. Then, using a protractor and a ruler, draw radii forming 5 equal angles at the center of the circle. How many degrees will there be in each angle? Draw lines connecting the points at which the radii meet the circumference. You will have a **pentagon,** a figure with 5 sides. Extend the sides of the pentagon as partly shown in the figure at the right. You will have a 5-point star.



Work the exercises in column A. If you fail on any exercise, do the exercise opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Test	C Review
1. Constructing Lines and Angles		
Using the compasses: Bisect a vertical line.	Using the compasses: Bisect an oblique line.	517
Draw a line perpendicular to a 3″ line, MN, $1\frac{1}{4}$″ from the right end.	Draw a line perpendicular to a $2\frac{1}{2}$″ line, AB, 1″ from the left end.	517
Draw an obtuse angle equal to another obtuse angle, and bisect it.	Draw an acute angle equal to another acute angle, and bisect it.	518
Draw two vertical parallel lines $1\frac{1}{4}$″ apart.	Draw two oblique parallel lines $1\frac{1}{2}$″ apart.	518
2. Constructing Geometric Figures		
Using the compasses, draw: A rectangle $2\frac{3}{4}$″ by $1\frac{1}{2}$″.	Using the compasses, draw: A rectangle 3″ by $2\frac{1}{4}$″.	519
A parallelogram having a $2\frac{1}{4}$″ base and slant sides $1\frac{1}{4}$″ long.	A parallelogram having a $2\frac{3}{4}$″ base and slant sides $1\frac{1}{2}$″ long.	519
A triangle having sides $2\frac{3}{4}$″, $3\frac{1}{4}$″, and 3″ long.	A triangle having sides 3″, $2\frac{1}{2}$″, and $3\frac{1}{4}$″ long.	519
A regular hexagon having sides $1\frac{1}{8}$″ long.	A regular hexagon having sides $\frac{3}{4}$″ long.	521
Using a protractor, draw: A triangle with sides $2\frac{1}{4}$″ and $1\frac{3}{4}$″ forming a 64° angle.	Using a protractor, draw: A triangle with sides $1\frac{5}{8}$″ and $2\frac{1}{2}$″ forming a 49° angle.	520

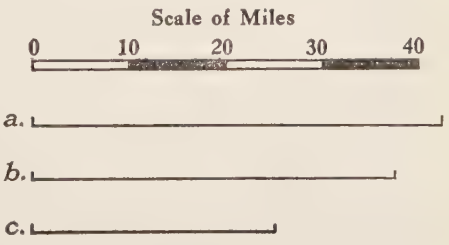
If you have ever made a model airplane or a model ship, the plans you used were **scale drawings** in which certain lengths stood for actual lengths. Most maps and working plans for houses, machines, etc., are scale drawings.

In scale drawings, we are able to represent measurements and distances accurately in a convenient space. The scale $1'' = 10'$ on a plan means that $1''$ in the drawing stands for $10'$ in the real object. On maps, an inch may stand for 1 mi., 10 mi., or 100 mi., according to the scale used. In dictionaries, you find that small objects are usually drawn to **multiplication scales.** The multiplication scale $\times 5$ means that the measurements in the drawing are 5 times the actual measurements of the object. Large objects are drawn to **fraction scales.** The fraction scale $\frac{1}{8}$ means that the drawing measurements are $\frac{1}{8}$ the actual measurements.

Find the missing values in the following:

	Scale	Scale Length	Actual Length
1.	$1'' = 16'$	$2\frac{1}{4}''$	?
2.	$\times 6$	?	$\frac{1}{8}''$
3.	$\frac{1}{20}$	$\frac{3}{4}''$	?
4.	$\times 4$	$\frac{5}{8}''$	?
5.	$\frac{3}{4}'' = 20'$	?	$50'$
6.	$\frac{1}{15}$	?	$7\frac{1}{2}''$

7. For how many miles does $1''$ stand on the road-map scale at the right? $\frac{1}{2}''$? $\frac{1}{4}''$? $1\frac{1}{2}''$? According to this scale, what distances are represented by lines *a*, *b*, and *c*?



8. For how many miles does $1''$ stand on this scale? Using this scale, draw lines representing 150 mi.; 250 mi.; 600 mi.

The *perimeter* of any plane figure is the distance around the figure and is equal to the sum of its sides. Before the perimeter of a figure can be found, the lengths of all the sides must be in the same unit of measure.

1. Does the perimeter of a square equal 4 times the length of one side? Why?

2. How do you find the perimeter of a rectangle?

Find the perimeters of the following:

	Figure	Sides		Figure	Sides
3.	Square	7 rd.	5.	Rectangle	17″ and 4′
4.	Triangle	6′, 8′, 42″	6.	Trapezoid	$7\frac{1}{2}$′, 78″, 17′, 21′

FINDING AREAS

The surface enclosed by the sides of a plane figure, or the *area* of the figure, is the number of units of *square measure* it contains.

SURFACE OR SQUARE MEASURE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet (sq. ft.)	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards (sq. yd.)	= 1 square rod (sq. rd.)
160	square rods (sq. rd.)	= 1 acre (A.)
640	acres (A.)	= 1 square mile (sq. mi.)

Review the above table; then work the following:

1. 2880 sq. in. = ? sq. ft.
2. 920 sq. rd. = ? A.
3. 603 sq. ft. = ? sq. yd.
4. $5\frac{1}{4}$ sq. ft. = ? sq. in.
5. 124 sq. rd. = ? sq. yd.
6. 2720 A. = ? sq. mi.

Many materials are sold by square measure. Carpet is sold by the square yard; wire screening, by the square foot. In order to know how much carpet to buy to cover a floor, or screening to cover a window, we need to know the area of the floor or the window. Knowing how to find areas is useful in many other ways.

The measurements of a figure are called its *dimensions*. Before the area of a figure can be found, its dimensions must be expressed in the same unit of measure.

Area of a rectangle = length × width Formula: $A = lw$

What is the area of a rectangle 80′ by 24′? Substituting the known values in the formula $A = lw$, we have $A = 80 \times 24$, or 1920. The area of the rectangle is 1920 sq. ft.

7. Find the area of a rectangle 37 yd. by 42 ft.

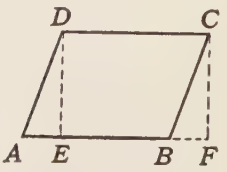
If the dimensions of a rectangle are given as base and height, the formula for the area is $A = bh$.

The area of a square having 6″ sides is 6×6, or 36 square inches. When we multiply a number by itself, we **square** the number. $6 \times 6 = 36$ may be written $6^2 = 36$. The small figure ^{2}, called an **exponent,** shows how many 6's have been multiplied together. $6^2 = 36$ is read **6 squared equals 36.** The square of 6 is 36.

Area of a square = side × side, or side squared Formula: $A = s^2$

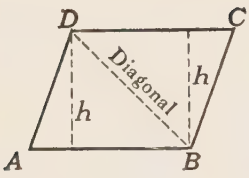
8. What is the area of a square having a side of 9 rd.?

Study the figure at the right. If you were to cut out triangle AED and place it in the position BFC, what kind of figure would $EFCD$ be? How would you find its area?



Area of a parallelogram = base × height Formula: $A = bh$

9. Find the area of a parallelogram having a base of 6′ and a height of 18″.

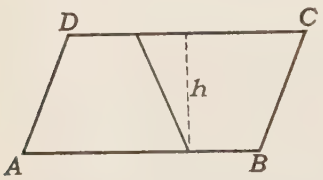


What is figure *ABCD?* How do you find its area? When you divide this figure into two equal parts by a **diagonal** line as shown, what figure is each part? You see that the area of a triangle is $\frac{1}{2}$ the area of a parallelogram having the same base and height.

Area of a triangle = $\frac{1}{2}$ base × height

Formula: $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

10. What is the area of a triangle having a base of 12 rd. and a height of 9 rd.?



This figure shows two equal trapezoids fitted together. What figure do they form? Notice that the base of the parallelogram equals the two bases of one trapezoid, or base 1 + base 2. Its height is the same as the height of either trapezoid. Is the area of each trapezoid equal to $\frac{1}{2}$ the area of the parallelogram?

Area of a trapezoid = $\frac{1}{2}$ sum of bases × height

Formula: $A = \frac{1}{2}(b1 + b2)h$ or $A = \frac{(b1 + b2)h}{2}$

11. Find the area of a trapezoid having bases of 9′ and 15′ and a height of 12′.

SCALES, PERIMETERS, AND AREAS

1. A map measuring 8″ × 6″ is drawn to a scale of 50 mi. to the inch. What actual area is represented on the map?

2. The perimeter of a square field is 200 rd. What is the length of 1 side? What is the area of the field?

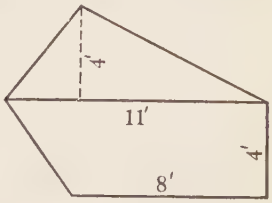
3. A city block shaped like a trapezoid has bases 300′ and 200′ long and a height of 180′. What is its area?

4. At \$1.10 a square yard, how much will it cost to cover with linoleum a floor 18′ × 14′?

5. If it costs \$36 to lay a walk 3 ft. wide and 40 ft. long, what is the cost per square foot?

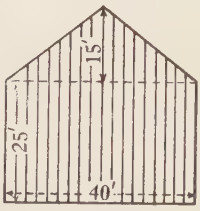
6. A triangular plot has a base of 60′ and a height of 30′. At 35¢ a pound, what will be the cost of grass seed for the plot if 1 lb. is used for each 150 sq. ft.?

The area of an irregular figure having straight sides often may be found by dividing the figure into rectangles, triangles, and other simple geometric figures. The total area of the irregular figure is the sum of the areas of the figures into which it is divided.



7. Find the total area of the irregular figure at the right.

8. The front and the back of a barn are each shaped like the pentagon at the right. The rectangular part measures 40′ × 25′. The triangular section has a base of 40′ and a height of 15′. Each side of the barn measures 80′ × 25′. (a) What is the total area of the four sides? (b) If paint costs \$1.45 a gallon, and 1 gallon will cover 300 sq. ft. two coats, how much will it cost to paint the four surfaces two coats?



9. Measure the length and the width of your schoolroom. Using the scale $\frac{1}{2}'' = 8'$, draw a floor plan of the room.

530 FINDING THE CIRCUMFERENCE AND THE AREA OF A CIRCLE

By careful measurement, it has been found that the circumference of any circle is about $3\frac{1}{7}$ or 3.14 times the diameter. More accurately, it is 3.1416 times the diameter. The Greek letter π (pī) is used to express 3.1416, 3.14, or $3\frac{1}{7}$. Since the diameter of a circle is twice the radius, the rule for finding the circumference of a circle is:

> **Circumference of a circle = $\pi \times$ the diameter or $\pi \times$ twice the radius**
> **Formula: $C = \pi d$ or $C = 2\pi r$**

Use $3\frac{1}{7}$ for π unless otherwise directed.

Find the circumferences of circles having these radii:

1. 2 ft. **2.** 12 in. **3.** $1\frac{1}{2}$ yd. **4.** 15 in. **5.** 20 ft.

Since the circumference of a circle $= \pi \times$ the diameter,

> **Diameter of a circle = circumference $\div \pi$**
> **Formula: $d = \frac{C}{\pi}$**

Find the diameters of circles having these circumferences:

6. 11 ft. **7.** 121 in. **8.** 132 rd. **9.** 220 ft. **10.** 154 yd.

> **Area of a circle = $\pi \times$ the radius squared**
> **Formula: $A = \pi r^2$**

Find the areas of circles having these radii:

11. $3\frac{1}{2}$ in. **12.** 14 ft. **13.** 21 yd. **14.** 32 rd.

15. A man wants to rope off a piece of ground in the form of a circle for use as a show ring at a fair. The diameter of the ring is to be 28′. How many feet of rope are needed, allowing 5 extra feet for knots?

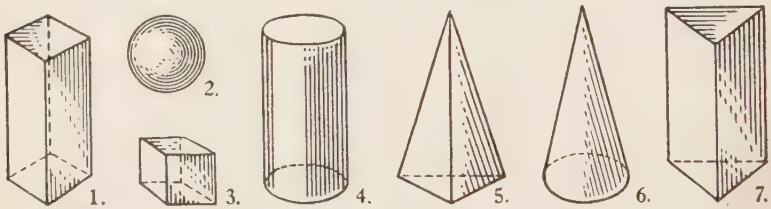
Work the exercises in column A. If you fail on any exercise, do the exercise opposite it in column B. If you still make mistakes, review the pages in column C.

A Test			B Drill			C Review
1. Working with Scales						
Scale	Scale Length	Actual Length	Scale	Scale Length	Actual Length	
$1'' = 24$ mi.	$3\frac{1}{8}''$	?	$1'' = 80$ mi.	$3\frac{3}{4}''$	?	525
$\frac{3}{8}'' = 12'$	?	$60'$	$\frac{3}{4}'' = 30'$	?	$100'$	
$\times 6$	?	$\frac{3}{4}''$	$\times 3$	?	$\frac{1}{12}''$	
$\frac{1}{16}$	$2\frac{1}{4}''$	?	$\frac{1}{12}$	$2\frac{1}{2}''$	?	
2. Finding Perimeters						
Find the perimeter of:			Find the perimeter of:			526
A rectangle, 6′3″ by 5′			A rectangle, 12″ by $3\frac{1}{2}''$			
A triangle, sides 9″, 12″, and $6\frac{1}{4}''$			A triangle, sides $6\frac{1}{2}'$, 8′, and 9′			
A trapezoid, sides 80′, 41′, $42\frac{3}{4}'$, 16′			A trapezoid, sides 21″, 18″, 12″, $16\frac{3}{4}''$			
3. Finding Areas of Plane Figures						
Find the area of:			Find the area of:			
A rectangle, $4\frac{3}{4}''$ by 16″			A rectangle, $5\frac{1}{2}'$ by 12′			527
A square, sides $8\frac{1}{2}'$			A square, sides 16 yd.			527
A parallelogram, base $24\frac{1}{4}''$, height 16″			A parallelogram, base $15\frac{1}{2}'$, height 6′			527
A triangle, base 12′, height $4\frac{1}{2}'$			A triangle, base 8 yd., height $5\frac{1}{2}$ yd.			528
A trapezoid, bases 36″ and 48″, height 18″			A trapezoid, bases 80′ and 60′, height 24′			528
4. Finding the Dimensions and Areas of Circles						
Radius	Diam.	Circum.	Area	Radius	Diam.	Circum.	Area	
35″	?	?	?	28′	?	?	?	530
?	?	264′	?	?	?	176 yd.	?	

A line has one dimension—**length.** A rectangular surface has two dimensions—**length** and **width.** Many figures have three dimensions—**length, width,** and **height**—which is sometimes called **depth** or **thickness.**

A figure which has three dimensions is a *solid*.

The surfaces of solids are plane figures or curved surfaces or a combination of both.



Name objects that are shaped like each of the above figures. Find pictures of such objects. Tell whether the objects are made in these shapes from necessity, for convenience, or because they look better in these forms.

A solid whose six surfaces are all rectangles is called a *rectangular solid*. If each rectangular surface is a square, the solid is called a *cube*.

1. Which of the above figures are rectangular solids? Which is a cube?

A solid whose two bases are parallel and equal and whose sides are parallelograms is a *prism*. A prism is named according to the shape of its bases, as a *rectangular prism* or a *triangular prism*.

2. Which of the above figures are rectangular prisms? Which is a triangular prism?

A solid having two bases which are equal and parallel circles connected by a uniformly curved surface is a *cylinder*.

3. Which of the above figures is a cylinder?

4. Most canned goods come in cylindrical containers. Can you tell why?

5. Why are newspapers printed on cylindrical rolls rather than from flat sheets of type?

A solid which has a circular base and a curved surface that tapers to a point or *vertex* is a *cone*.

6. Which of the figures on the opposite page is a cone?

A solid whose sides are triangles which meet in a common point or *vertex* is a *pyramid*. The base of a pyramid may be a rectangle, a triangle, or some other straight-line figure.

7. Which of the figures on the opposite page is a pyramid?

A solid which has a curved surface every point of which is equally distant from a point within called the center is a *sphere*.

8. Which of the figures on page 532 is a sphere?

9. Ball bearings used in machinery are spheres. Ask some automobile mechanic to tell you how and why ball bearings are used in certain automobile parts.

COMPARING PLANE AND SOLID FIGURES

In what ways are the following figures alike, and in what ways are they different:

1. A square and a cube?

2. A rectangle and a rectangular prism?

3. A triangle and a pyramid?

4. A circle and a cylinder?

5. A circle and a sphere?

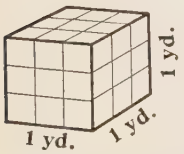
6. Does a solid occupy space? Is this also true of a flat surface? Why?

A cube 1″ long, 1″ wide, and 1″ high is a **cubic inch (cu. in.)**. What is a **cubic foot (cu. ft.)**? A **cubic yard (cu. yd.)**?

If you were to take a box whose inside dimensions were $1' \times 1' \times 1'$ and fill it with 1-inch cubes, you would find that it would hold exactly 12 layers of inch cubes and that in each layer there would be 12 rows each containing 12 cubic inches. In one layer, therefore, there would be 12×12 cubic inches, or 144 cubic inches. In the 12 layers, there would be 12×144 cu. in., or 1728 cu. in.

1 cu. ft. $= 12 \times 12 \times 12$ cu. in., or 1728 cu. in.

When we multiply any three equal numbers together, as $12 \times 12 \times 12$, the product is called the **cube** of the number. The cube of 12 may be written $12 \times 12 \times 12 = 1728$, or $12^3 = 1728$.



The figure at the left represents a cubic yard divided into cubic feet. How many cubic feet are there in one row of the top layer? In the layer? How many cubic feet in all the layers? $3 \times 3 \times 3$ cu. ft. = 27 cu. ft. in a cubic yard.

Solids are measured in cubic inches, cubic feet, or cubic yards. The number of cubic units in a solid is called its *volume*, or *capacity*.

Learn this table of cubic measure:

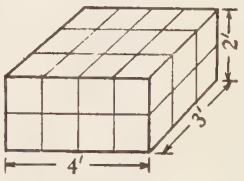
CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.) = 1 cubic yard (cu. yd.)

Volume of a cube = edge × edge × edge, or edge cubed
Formula: $V = e^3$

1. What is the volume of a cube $6'' \times 6'' \times 6''$? Of a cube measuring $1\frac{1}{2}' \times 1\frac{1}{2}' \times 1\frac{1}{2}'$?

The *height* or *altitude* of a prism is the perpendicular distance between its bases.



The figure at the right represents a rectangular prism 4 ft. by 3 ft. by 2 ft. divided into cubic feet. How many cubic feet are there in the prism? Multiplying the number of cubic units in one row by the number of rows by the number of layers gives 4 cu. ft. $\times$ 3 $\times$ 2, or 24 cu. ft.

A short way to find the volume of a rectangular prism is to multiply the length by the width by the height. Using V for volume, l for length, w for width, and h for height, we have as the formula for finding the volume of a rectangular prism $V = lwh$.

Since $l \times w$ gives the area of a base of a rectangular prism, we can multiply the area of one base by the height to find the volume of the prism. The volume of any other kind of prism can be found in the same way.

How to Find the Volume of Any Prism

A prism 8″ high has triangular bases each of which has a height of 6″ and a base of 5″. What is the volume of the prism?

$\frac{1}{\cancel{2}} \times \cancel{6}^{3} \times 5 = 15$

Find the area of one base of the prism.

$15 \times 8 = 120$

To find the volume of the prism, multiply the area of the base by the height of the prism.

The volume of the prism is 120 cu. in.

Volume of any prism = area of base (B) $\times$ height
Formula: $V = Bh$

Find the volumes of these prisms:

	Area of Base	Height		Area of Base	Height
1.	18 sq. in.	4 in.	**3.**	5 sq. yd.	$2\frac{1}{2}$ yd.
2.	25.5 sq. ft.	7 ft.	**4.**	$7\frac{1}{2}$ sq. ft.	9 in.

Find the volumes of these rectangular prisms:

	Base	Height		Base	Height
5.	5 in. $\times$ 12 in.	16 in.	**8.**	9 in. $\times$ 6 in.	2 ft.
6.	2 ft. $\times$ $1\frac{3}{4}$ ft.	4 ft.	**9.**	3 yd. $\times$ 1 ft.	4 yd.
7.	$5\frac{1}{2}$ ft. $\times$ 4 ft.	$8\frac{1}{2}$ ft.	**10.**	2′3″ $\times$ 2′6″	3′6″

11. A prism 6′ high has triangular bases. Each triangle has a height of 3′ and a base of 4′. Find the volume of the prism.

12. A schoolroom 24′ by 30′ is 12′ high. The class attendance, including the teacher, is 30. Find the cubic feet of air space per person.

13. Find the capacity in cubic feet of a goldfish tank having inside measurements of 2′6″ $\times$ 18″ $\times$ 2′.

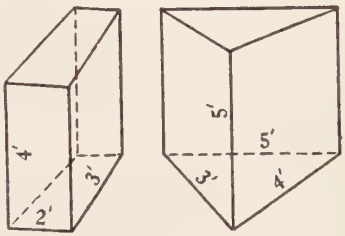
14. A cubic foot of ice weighs 57.5 lb. What is the weight of 600 blocks of ice each 4′ $\times$ 2′ $\times$ 1′?

15. Mr. Wilson has a grain bin that is 4 ft. deep, 18 ft. long, and 5 ft. wide. Allowing .8 bu. of grain to 1 cu. ft., how many bushels will the bin hold when full?

16. Mr. Wilson's coal bin is 7′ wide and $9\frac{1}{2}$′ long. How many tons of hard coal does it contain when filled to a depth of 5′? (1 T. = 35 cu. ft.)

17. A watering trough 8′ long has triangular ends. Each end has a base of 18″ and an altitude of 12″, inside dimensions. How many cubic feet of water can the trough hold?

The sides of a prism are called its **lateral surfaces.** As each side of a prism is a rectangle, we can find the **lateral area** of a prism by finding the sum of the areas of all the sides. An easier way is to multiply the perimeter of one base by the height. Why?



How to Find the Lateral Area of a Prism

A packing case 4′ high has a base measuring $3' \times 2'$. What is its lateral area?

$2' + 2' + 3' + 3' = 10'$ Find the perimeter of the base of the prism.

$10 \times 4 = 40$ To find the lateral area of the prism, multiply the perimeter of the base by the height.

The lateral area of the case is 40 sq. ft.

To find the area of all the surfaces of a prism, add the area of the two bases to the area of the sides.

1. At \$.75 a square yard, find the cost of plastering the walls and ceiling of a room 26′ long, 16′ wide, and 8′ high, deducting 16 sq. yd. for doors and windows.

2. A rectangular concrete pillar has a base $3\frac{1}{4}' \times 3\frac{1}{2}'$ and is 14′ high. How many cubic feet of concrete does it contain? What is the lateral area of the pillar?

3. A box forms a triangular prism $1\frac{1}{2}'$ deep. The base of the prism has sides 10″, 10″, and 10″ and an altitude of 8″. How many square feet of material are needed to line the box, including the cover?

The bases of a cylinder, like those of a prism, are the same size and shape. You can therefore find the volume of a cylinder in the same way that you find the volume of a prism—by multiplying the area of one base by the height of the cylinder.

Volume of a cylinder = area of base × height
Formula: $V = Bh$, or $V = \pi r^2 h$

How to Find the Volume of a Cylinder

A cylindrical water tank is 7′ high and 4′ in diameter. What is the volume of the tank in cubic feet?

$\frac{1}{\cancel{2}} \times \cancel{4}^{2} = 2'$ Find the radius of the cylinder.

$\frac{22}{7} \times 2 \times 2 = \frac{88}{7}$

$\frac{88}{\cancel{7}} \times \cancel{7} = 88$ Find the area of the circular base of the cylinder, using the formula $A = \pi r^2$.

To find the volume of the cylinder, multiply the area of the base by the height.

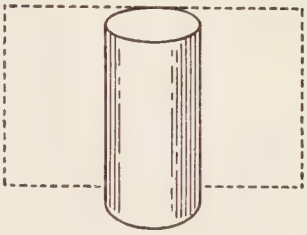
The volume of the tank is 88 cu. ft.

Find the volume of each of these cylinders, using $3\frac{1}{7}$ for π:

	Radius	Height		Diameter	Height
1.	8 in.	12 in.	3.	12 ft.	15 ft.
2.	16 ft.	14 ft.	4.	8 ft.	10 ft. 6 in.

5. A tank is 4 ft. high and 8 ft. in diameter. Allowing $7\frac{1}{2}$ gal. to the cubic foot, how many gallons of water does the tank hold when full? Use $\pi = 3.14$.

6. How much will it cost to fill with fodder a cylindrical silo 9′ in diameter and 21′ high at 3¢ a cubic foot?



This diagram shows how the lateral surface of a cylinder would look if spread out flat. What plane figure does it form? Is the base of the rectangle equal to the circumference of the cylinder? Is the height of the rectangle equal to the height of the cylinder? Since the area of a rectangle equals base $\times$ height, how can you find the lateral area of a cylinder?

How to Find the Lateral Area of a Cylinder

The base of a cylindrical can has a radius of $3\frac{1}{2}''$. The height of the can is $12''$. What is the lateral area of the can?

$\cancel{2} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} = 22$

$22 \times 12 = 264$

Find the circumference of the base of the cylinder. ($2\pi r$).

To find the lateral area of the cylinder, multiply the circumference of the base by the height of the cylinder.

The lateral area of the can is 264 sq. in.

Lateral area of a cylinder = circumference of base ($2\pi r$ or πd) $\times$ height.

Formula: $A = 2\pi rh$, or πdh

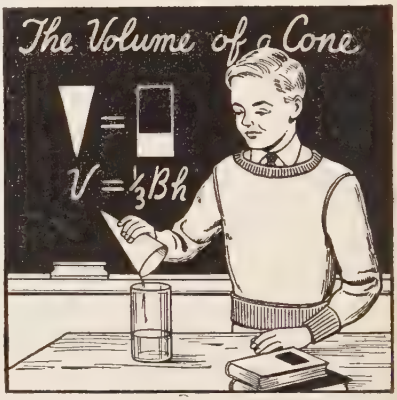
To find the *total* area of a cylinder, add the area of the two circular bases to the area of the lateral surface.

Find the lateral areas and the total areas of cylinders having these dimensions:

	Radius of Base	Height		Diameter of Base	Height
1.	3 in.	8 in.	3.	$2''$	$18''$
2.	14 ft.	50 ft.	4.	$8''$	$2'$

The *height* or *altitude* of a cone is the perpendicular distance from the vertex to the base.

If you should fill a cone-shaped drinking cup with water and then empty the water into a cylindrical drinking glass having the same base and height as the cone, you would find that the glass would be exactly $\frac{1}{3}$ full. The volume of any cone is $\frac{1}{3}$ the volume of a cylinder having the same base and height.



How to Find the Volume of a Cone

A cone-shaped tent is 12′ in diameter at the base and 8′ high. How many cubic feet of air does it contain?

$\frac{1}{2} \times 12' = 6'$ radius of base

$\frac{22}{7} \times 6 \times 6 = \frac{792}{7}$ area of base

$\frac{1}{3} \times \frac{264}{792} \times 8 = \frac{2112}{7} = 301\frac{5}{7}$

Find the area of the base of the cone.

To find the volume of the cone, find $\frac{1}{3}$ of the area of the base times the height.

The tent contains $301\frac{5}{7}$ cu. ft. of air.

Volume of a cone = $\frac{1}{3}$ area of base × height
Formula: $V = \frac{1}{3}Bh$, or $\frac{1}{3}\pi r^2 h$

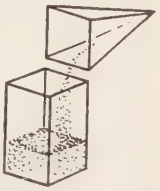
1. The diameter of the base of a cone-shaped sand pile is 9′, and the height is 7′. Find the number of cubic feet of sand in the pile.

*Optional material.

Find the volumes of cones having these dimensions:

	Radius of Base	Height		Diameter of Base	Height
2.	14 ft.	10 ft.	4.	21 ft.	18 ft.
3.	8 in.	12 in.	5.	20 in.	14 in.

The *height* or *altitude* of a pyramid is the perpendicular distance from the vertex to the base.



By experimenting with a hollow pyramid, an empty box having the same base and height as the pyramid, and some sand, you would discover that the volume of a pyramid is $\frac{1}{3}$ the volume of a prism having the same base and height. You can therefore find the volume of a pyramid in the same way that you find the volume of a cone — by finding $\frac{1}{3}$ of the area of the base $\times$ the height.

Volume of a pyramid = $\frac{1}{3}$ area of base $\times$ height
Formula: $V = \frac{1}{3}Bh$

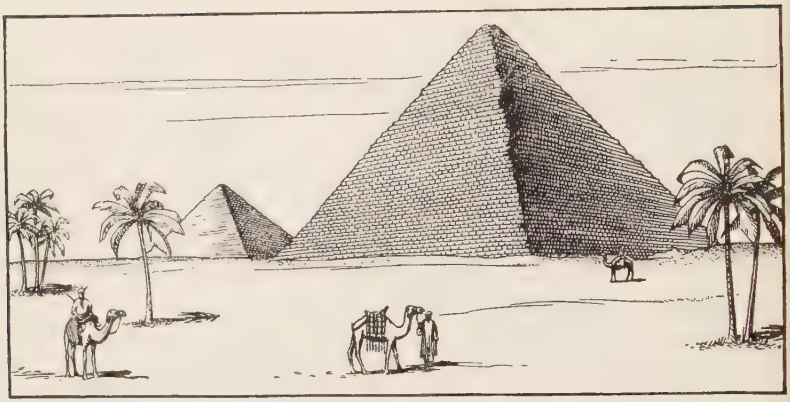
Find the volumes of pyramids having these dimensions:

	Area of Base	Height		Area of Base	Height
6.	24 sq. in.	7 in.	8.	49 sq. in.	13 in.
7.	40 sq. ft.	11.5 ft.	9.	17 sq. ft.	9.5 ft.

10. In a grain elevator, wheat is drawn from the bottom of a bin through a pyramid-shaped chute 4′ square at the top and 4′6″ deep. If 1 bu. = $1\frac{1}{4}$ cu. ft., how many bushels of wheat does the chute hold?

11. A pyramid 14″ high has a base shaped like a triangle. The triangle has a height of 8″ and a base of 6″. What is the volume of the pyramid?

*Optional material.



Two famous pyramids—tombs of ancient Egyptian kings.

The most famous pyramids in the world are on the banks of the Nile River. They were built of huge blocks of stone dragged into place by armies of slaves.

1. The oldest of the Egyptian pyramids was built about 3000 B.C. About how many years ago was that?

2. The largest of the Egyptian pyramids is the Great Pyramid. A Greek historian of ancient times stated that it took 100,000 men 20 years to build this pyramid. This is the same as the work of how many men for 1 year?

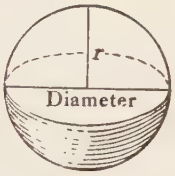
3. The Great Pyramid is estimated to weigh 6,848,000 T. and to contain 2,300,000 blocks of stone. What is the average weight of each block of stone?

4. The original height of the Great Pyramid was 481 ft., but wind and weather have worn away the rock until now the pyramid is 451 ft. high. About what per cent has the pyramid decreased in height?

***5.** The area of the base of the Great Pyramid is 6,292,000 sq. yd., and the height is 451 ft. Find the volume of the pyramid in cubic yards.

*Optional.

The *radius* of a sphere is the distance from its center to any point on its surface. The *diameter* of a sphere is twice the radius.



Take a wooden ball and cut it into two equal parts, or hemispheres. Notice that the cut surface of each hemisphere is a circle having the same radius as the sphere.

Tack one end of a cord at the exact center of the flat circular surface of one hemisphere. Then wind the cord so as to cover the surface, as shown above. Cut off the rest of the cord. Unwind the cord, and measure its length.

In the same way, wind a similar piece of cord around the outside curved surface of the other hemisphere, as shown in the second figure in the drawing. Unwind and measure the length of cord used. It should be almost exactly twice the length of the first cord. This shows that the area of the curved surface of a hemisphere is twice the area of a circle having the same radius. Thus, the area of the surface of an entire sphere is four times the area of a circle having the same radius as the sphere.

Since the area of a circle $= \pi r^2$, the area of a sphere is 4 times πr^2.

How to Find the Area of a Sphere

Find the area of a globe which has a radius of 3″.

$3 \times 3 = 9$ Find the square of the radius.

$9 \times 4 \times \frac{22}{7} = \frac{792}{7} = 113\frac{1}{7}$ To find the area of the sphere, multiply the square of the radius by 4 times π.

The area of the globe is $113\frac{1}{7}$ sq. in.

*Optional material.

Area of a sphere = $4\pi \times$ the radius squared
Formula: $A = 4\pi r^2$

Find the areas of spheres having these dimensions:

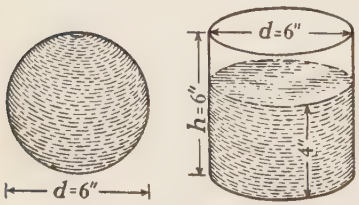
1. Radii: 7″; 5′; $4\frac{1}{2}$″; 10 yd.; 84 ft.; 3 ft. 6 in.

2. Diameters: 21″; 8′; 13 yd.; 4 ft. 6 in.; 2′ 8″

3. A building has a dome shaped like a hemisphere with a radius of 21 ft. Find the area of the dome in square yards.

4. The earth, which is nearly a perfect sphere, has a diameter of about 8000 mi. Find the area of the earth, rounded to the nearest hundred-thousand square miles.

FINDING THE VOLUME OF A SPHERE*



If you filled with water a round glass globe having an inside diameter of 6″, and then poured the water into a cylindrical can 6″ high and having a diameter of 6″, you would find that the can would be about $\frac{2}{3}$ full. You can thus see that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder having the same height and diameter.

Notice in the above cylinder that the height equals the diameter; that is, $h = 2r$. The volume of the cylinder = $\pi r^2 h$. Substituting $2r$ for h, we have $\pi r^2 \times 2r$, or $2\pi r^3$. Since the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder, it is $\frac{2}{3} \times 2\pi r^3$, or $\frac{4}{3}\pi r^3$.

Volume of a sphere = $\frac{4}{3} \times \pi \times$ the radius cubed
Formula: $V = \frac{4}{3}\pi r^3$

*Optional material.

How to Find the Volume of a Sphere

How many cubic inches of silver are there in a silver ball having a radius of 2″?

$V = \frac{4}{3}\pi r^3$ Write the formula for the volume of a sphere.

$r^3 = 2 \times 2 \times 2 = 8$ Find the cube of the radius.

$V = \frac{4}{3} \times \frac{22}{7} \times 8 = \frac{704}{21} = 33\frac{11}{21}$ To find the volume of a sphere, substitute the known values for the letters in the formula, and multiply.

The ball contains $33\frac{11}{21}$ cu. in. of silver.

1. Find the volumes of spheres having these radii: $3\frac{1}{2}'$; 4″; 9″; $4\frac{1}{2}$ ft.; 12 ft.; 17 yd.; $5\frac{1}{2}$″; 2′6″; 1′9″.

2. A granite ball 12″ in diameter forms the top of a monument. What is the volume of the ball?

3. A sphere having a diameter of 8″ is cut from an 8″ stone cube. How many cubic inches of stone are cut away?

4. Mary bought a spherical goldfish bowl which had an inside diameter of 14″. If 1 gal. = 231 cu. in., about how many gallons will the bowl hold when full of water?

5. A spherical balloon 60 ft. in diameter is to be filled with gas. How many cubic feet of gas will it hold?

6. A ball of lead 10″ in diameter is melted and the lead used to make 2-in. cubes. About how many of the cubes can be made from the ball of lead?

7. Compare the volumes of a 6-in. cube and a sphere 6″ in diameter.

*Optional material.

Work the exercises in column A. If you fail on any exercise, do the exercise opposite it in column B. If you still make mistakes, review the pages in column C.

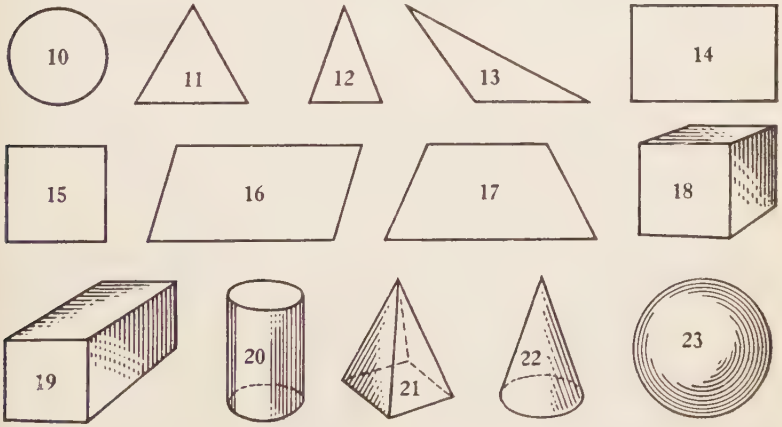
A Test	B Test	C Review Pages
1. Finding the Volume and the Lateral Area of a Prism		
Rectangular prism: Base, $5'' \times 2\frac{1}{2}''$; height, $5''$ Triangular prism: Triangles — base $12'$; height $8'$; sides $12'$, $10'$, $10'$; height of prism, $16'$	Rectangular prism: Base, $4'' \times 12''$; height, $16''$ Triangular prism: Triangles — base $6''$; height $4''$; sides $6''$, $5''$, $5''$; height of prism, $12''$.	535-537
2. Finding the Volume and the Lateral Area of a Cylinder		
Radius, $6''$; height, $21''$ Diameter, $16'$; height, $60'$	Radius, $4'$; height, $14'$ Diameter, $7''$; height, $40''$	538-539
3. Finding the Volumes of Cones and Pyramids*		
Cone: Radius of base, $3\frac{1}{2}''$; height, $12''$ Cone: Diameter of base, $3'$; height, $9'$ Pyramid: Base a $5''$ square; height, $8''$ Pyramid: Base a triangle having a base of $6''$ and an altitude of $4\frac{1}{2}''$; height of pyramid, $9''$	Cone: Radius of base, $7''$; height, $18''$ Cone: Diameter of base, $14'$; height, $15'$ Pyramid: Base a $4'$ square; height, $6'$ Pyramid: Base a triangle having a base of $6'$ and an altitude of $5'$; height of pyramid, $8'$	540-541
4. Finding the Area and the Volume of a Sphere*		
Radius, $15''$ Diameter, $6'$	Radius, $9''$ Diameter, $4'$	543-545

*Optional material.

Draw each of the following:

1. A vertical line
2. A curved line
3. A broken line
4. A horizontal line
5. Two parallel lines
6. An acute angle
7. An obtuse angle
8. A right angle
9. A line perpendicular to an oblique line

Give the correct name of each of the following, and tell whether it is a plane or a solid figure:



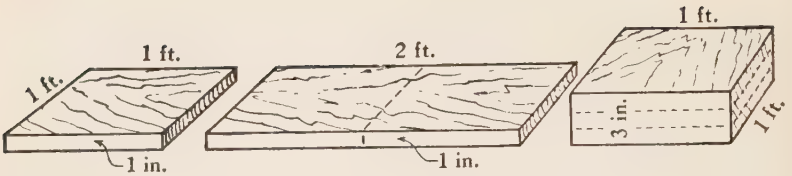
24. Write the formulas for finding the areas of figures 10, 11, 14, 15, 16, and 17, and the volumes of figures 18, 19, and 20.

Give a brief definition of each of the following:

25. Area
26. Volume
27. Angle
28. Vertex
29. Arc
30. Solid
31. Cube
32. Diameter
33. Sector
34. Circumference
35. Lateral surface of a cylinder

*36. Write the formula for finding the volume of a pyramid; a cone; a sphere.

*Optional material.



1 Board Foot 2 Board Feet 3 Board Feet

Lumber is usually measured by the **board foot (bd. ft.). A board foot is a piece of lumber 1′ long, 1′ wide, and 1″ thick.** If a board is 2′ long, 1′ wide, and 1″ thick, it contains 2 bd. ft.

If a board is more than 1″ thick, think of it as separated into layers 1″ in thickness, as in the third figure in the drawing above. A board 1′ long, 1′ wide, and 3″ thick contains 3 bd. ft.

Boards less than 1″ thick are considered as 1″ thick in measuring board feet.

How to Find the Number of Board Feet in Lumber

How many board feet are there in 24 pieces of lumber each 2″ thick, 4″ wide, and 12′ long?

Thickness = 2″
Width = 4″ = $\frac{1}{3}$′
Length = 12′

Express the thickness of each piece in inches, and the width and length in feet or fractions of a foot.

$2 \times \frac{1}{\cancel{3}} \times \overset{4}{\cancel{12}} = 8$

Find the product of these three numbers to find the number of board feet in one piece.

$8 \times 24 = 192$

Multiply the number of board feet in 1 piece by the number of pieces to find the number of board feet in all the pieces.

There are 192 bd. ft. in the 24 pieces.

*Optional material.

Find the number of board feet in the following:

1. 16 pieces, each $\frac{7}{8}''$ thick, 6″ wide, and 8′ long
2. 320 pieces, each 8″ thick, 12″ wide, and 12′ long
3. 48 pieces, each 2″ thick, 6″ wide, and 16′ long
4. 500 pieces, each 3″ thick, 4″ wide, and 14′ long

Some finished lumber, such as moldings and certain kinds of trim, is sold by the **linear foot,** or foot of length. Most other lumber, however, is sold by the thousand (M) board feet. A quoted price of \$30 per M for lumber means a charge of \$30 per 1000 bd. ft.

How to Find the Cost of Lumber

How much will 60 pieces of lumber each 1″ thick, 6″ wide, and 16′ long cost at \$45 per M?

$1 \times \frac{1}{2} \times \frac{16}{8} \times 60 = 480$ — Find the number of board feet.

$\$45 \div 1000 = \$.045$ — Divide the price per 1000 board feet by 1000 to find the price per board foot.

$\$.045 \times 480 = \$21.60$ cost of lumber

To find the cost of the lumber, multiply the price per board foot by the number of board feet.

In finding the cost of lumber, count any fraction of a board foot as 1 board foot.

5. Find the cost of 84 ft. of molding at $1\frac{1}{2}¢$ a linear foot.

6. At \$35 per M, what is the cost of 45 pieces of lumber 2″ thick, 4″ wide, and 14′ long?

Abbreviations are often used in bills for lumber. Thus, 62 pc. 1 × 6 × 16′ stands for 62 pieces, each 1″ thick, 6″ wide, and 16′ long.

7. What is the cost of 20 pc. 2 × 4 × 16′ @ \$42 per M?

*Optional material.

In looking up world records for track events, Jack found that distances are given in **meters.** The meter equals 39.37 inches, or about 1.1 yard. How does a 100-meter race compare in length with our 100-yard dash?

Centuries ago, trade throughout the world suffered because of the confusion caused by the use of different measures in different countries. Several units of measure were called by the same name, and one unit was often called by several names. At last, a group of French scientists worked out a new system of measures of length, surface, volume, capacity, and weight, which was called the **metric system.** The **meter** is the basic unit of the system and equals one ten-millionth of the distance from the Equator to the Pole.

The metric system is a decimal system; that is, each unit of measure in a table is either 10, 100, or 1000 times the next smaller unit and $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ the size of the next larger unit. This system is easy to use, and today is the standard system of measures in most of the countries of the world. In our country, the metric system is used in scientific work, in certain athletic events, and by exporters, importers, and customs authorities in figuring duties and prices.

Each table of the metric system has one standard unit. The standard unit of length is the **meter.** The names of the larger and smaller units in each table are formed by adding to the name of the standard unit the prefixes which are given below with their meanings:

Latin	**milli,** one thousandth	Greek	**deka,** ten
	centi, one hundredth		**hekto,** one hundred
	deci, one tenth		**kilo,** one thousand

Millimeter means $\frac{1}{1000}$ of a meter; **centimeter** means $\frac{1}{100}$ of a meter; **kilometer** means 1000 meters. What is the meaning of **decimeter? Dekameter? Hektometer?**

*Optional material.

METRIC TABLE OF LENGTH	
10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters (cm.)	= 1 decimeter (dm.)
10 decimeters (dm.)	= 1 meter (m.)
10 meters (m.)	= 1 dekameter (dkm.)
10 dekameters (dkm.)	= 1 hektometer (hm.)
10 hektometers (hm.)	= 1 kilometer (km.)

1. A radio station broadcasts on a radio wave length of 370 meters. About how long is this wave length in yards?

2. The distance by highway from Paris to Rome is 1490 km. About what is the distance in miles? (1 km. = $\frac{5}{8}$ mi.)

3. According to the above rule, 1 in. equals about how many centimeters?

4. Get a meter stick marked in centimeters. Find the width of your schoolroom in meters and the width of your desk in centimeters.

METRIC MEASURES OF AREA OR SURFACE*

The standard metric unit of area is the **square meter (sq. m.).** A square measuring 1 meter (10 decimeters) on each side is a square meter. It contains 10×10, or 100 square decimeters. 100 sq. dm. = 1 sq. m. (about 1.2 sq. yd.). In the metric table of surface measures, each unit is 100 times the next smaller unit.

1. Find in square meters the area of a lot 40.8 m. by 21 m.

*Optional material.

The **cubic meter** (1 m. $\times$ 1 m. $\times$ 1 m.) is the standard metric unit of volume. Since 1 m. = 10 dm., a cubic meter contains $10 \times 10 \times 10$, or 1000 cu. dm. 1000 cu. dm. = 1 cu. m. (about 1.31 cu. yd.). In the metric table of cubic measures, each unit is 1000 times the next smaller unit.

1. The excavation for a cellar is to be 10.5 m. by 14 m. by 3 m. How many cubic yards of earth must be removed?

The **liter** is the standard metric unit of capacity. It is used in measuring liquids. A can with inside measurements 10 cm. $\times$ 10 cm. $\times$ 10 cm. holds 1 liter **(l.),** or about 1.1 qt. In the metric table of capacity, each unit is 10 times the next smaller unit.

2. An importer received a shipment of 256 l. of perfume from France. How many quarts of perfume was this?

METRIC MEASURES OF WEIGHT*

The standard metric unit of weight is the **gram (g.).** One gram equals the weight of 1 cubic centimeter of water at a certain temperature. The metric unit of weight most commonly used is the **kilogram (kg.),** which equals 1000 grams (about 2.2 lb.). 1000 kg. = 1 metric ton (t.). In the metric table of weight, each unit is 10 times the next smaller unit.

1. In one year, France imported 80,000,000 kg. of copper from our country. How many pounds was this? How many tons?

2. A firm in Brazil shipped 900 metric tons of frozen beef to a customer in our country. The import duty was 6¢ a pound. How much duty did the customer pay?

3. Find the cost to an American importer of 1540 kg. of Edam cheese imported from Holland if the price per pound is 20¢ and the duty is 25% ad valorem.

*Optional material.

The finish of a 100-meter dash at the Olympic games.

The historic Greek custom of holding every four years an athletic festival known as the Olympic games was revived in 1896. The festival is held in a different country each time, and the best amateur athletes in the world compete. Because the metric system is used in so many countries, Olympic game distances are given in metric units.

The Olympic games of 1936 were held in Germany, and several new records were made by American athletes.

1. Express in feet and inches each of these record distances made by American athletes in the 1936 games, using 1 m. = 1.1 yd.:

a. Running high jump 2.03 m. **c.** Pole vault 4.35 m.

b. Running broad jump 8.06 m. **d.** Discus throw 50.48 m.

2. The 200-meter run was won by Jesse Owens in 20.7 seconds. Find the number of yards he ran per second.

3. A United States team won the 400-meter relay race in 39.8 seconds. At that speed, how long would it take the team to run a mile? Use 1 km. = $\frac{5}{8}$ mi.

*Optional material.

Try to work these problems without using pencil and paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. What is the perimeter of a 9″ square?

2. Find the perimeter in yards of a rectangle 15′ long and 6′ wide.

3. A sheet is $8\frac{1}{2}$ ft. long and 6 ft. wide. What is its area in square feet?

4. A triangular sail measures 9 ft. along the base and 8 ft. in height. Find its area.

5. The inside dimensions of a tank are 4 ft. by 2 ft. by $2\frac{1}{2}$ ft. How many gallons of water will the tank hold? (1 cu. ft. = $7\frac{1}{2}$ gal.)

6. The area of the base of a cylindrical silo is 61 sq. ft. The silo is 20 ft. high. Find its volume.

7. Find the area of a circular flower bed which has a radius of 7 ft.

8. A cube has edges each 5 in. long. Find the area of its six sides.

9. Find the lateral area of a cylinder if the radius of its base is 7″ and its height is 10″.

*10. In 2400 cm., there are how many meters?

*11. A liter equals about 1.1 liquid quarts. About how many quarts do 500 liters equal?

*12. Find the number of cubic feet in a pyramid having a base 6 ft. square, and a height of 20 ft.

*Optional material.

1. Write these numbers as Roman numerals: 1890, 1942.

2. Arrange in order from largest to smallest:

$\frac{5}{6}$ .875 $\frac{3}{4}$ $66\frac{2}{3}\%$ .899 .6 $3\frac{1}{2}$ 1.7%

3. The area of a square is 64″. What is the length of each side? What is the perimeter of the square?

4. The tax rate on property assessed at \$7500 is \$3.24 per \$100. What is the tax rate expressed as a per cent?

5. Find the annual premium on a life insurance policy for \$5000 at a rate of \$22.75 per \$1000.

6. Find the cost of \$8500 fire insurance on farm buildings for 1 yr. if the rate is 88¢ per \$100.

7. A stove is priced at \$89.50 cash or \$10 down and \$7.50 a month for 12 mo. About what per cent of increase is the installment price over the cash price?

8. Last year, Mr. Fisher's taxable balance was \$2367.15. What income tax did he pay, if the rate was 4%?

9. At $4\frac{1}{2}$¢ per kw-hr, find the cost of using an electric ironer for 3 hr. if the ironer uses 1200 watts an hour.

10. An importer paid duty of $2\frac{1}{2}$¢ per pound and 15% ad valorem on 5635 lb. of goods valued at \$225.40. Find the duty.

11. I paid \$182 a share for 6 shares of stock, and receive a yearly dividend of \$8 a share. What is my rate of income from the investment?

12. Find the cost of a \$1000 5% bond bought at $96\frac{1}{4}$. Disregard brokerage. What is the annual yield?

1. Divide the product of 6.254 and 6.05 by 3.12.

2. Multiply the sum of $3\frac{1}{4}$, $2\frac{1}{8}$, and $5\frac{1}{2}$ by $\frac{3}{5}$.

3. 15% of 4.8 = $\frac{1}{2}$% of ?

4. 3 bu. 3 pk. ÷ 6 = ?

5. 28.8 = ?% of 640

6. $5\frac{1}{4}$% of 36 = ?

7. Find the interest on \$648 for 33 da. at 5%.

8. Construct a triangle which has sides of 4″ and 2″ enclosing an angle of 45°.

9. A sheep ranch is 3 mi. square. How many rods of wire are needed to fence it with 6 strands of wire?

10. If a $\frac{1}{4}$-in. square represents an acre, how many acres will a 2-in. square represent?

11. A map drawn to a scale of 1″ = 10 mi. measures 5″ by 8″. How many square miles are represented by the map?

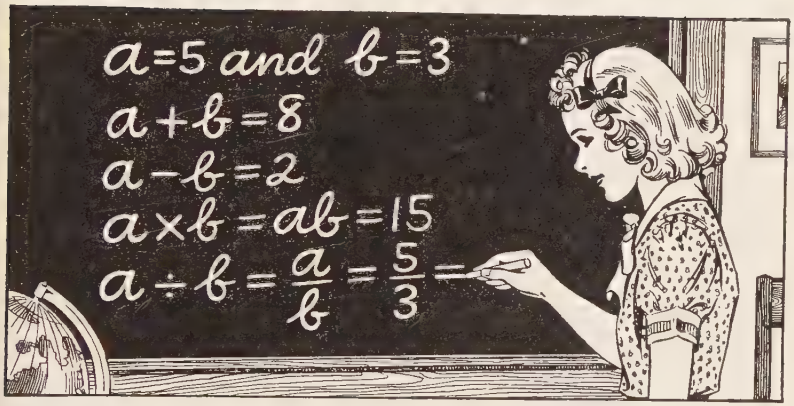
12. How many bushels of grain can a farmer store in a bin measuring 6′ by 6′ by 10′? (1 bu. = $1\frac{1}{4}$ cu. ft.)

13. A circular skating rink has a diameter of 50 ft. How many times as much skating surface is there in a circular rink having a diameter twice as great?

14. A cylindrical tank 5′ high has a diameter of 14″. How many gallons of water will it hold? (1 cu. ft. = $7\frac{1}{2}$ gal.)

15. At 20¢ a square foot, how much will it cost to paint the lateral surface of a cylindrical storage tank 20′ high and having a radius of 14′?

*Record your score on your Progress Chart. See page 321.



In Helen's problems, letters are used to stand for numbers.

If you study the illustration, you will see that Helen is working examples in which the letters a and b stand for the numbers 5 and 3. She has shown how one can express addition, subtraction, multiplication, and division when using letters in place of the numbers they stand for. How has she written the letters to show a multiplied by b? How has she written them to show a divided by b?

In **algebra,** letters are often used to stand for numbers. If you let the letter y stand for your age, $y + 2$ will represent your age in 2 years, and $y - 3$ was your age 3 years ago. $3 \times y$, or $3y$, will stand for 3 times your age; and $y \div 2$, $\frac{y}{2}$, or $\frac{1}{2}y$ will express $\frac{1}{2}$ your age.

Use letters and figures to write the following:

1. A number minus 8
2. Five more than a number
3. Three less than a number
4. A number multiplied by 17
5. One sixth of a number
6. 72 divided by a number

Almost the first arithmetic examples you ever worked were written with a question mark which stood for the number you were to find, as $3 + 2 = ?$, and $? - 2 = 3$. The question mark was not always in the same place in the examples, but you soon learned that it stood for the number you did not know. If you had used a letter in place of the question mark to stand for the unknown number, you would have been using algebra. Your examples would then have been written: $3 + 2 = n$, $n - 2 = 3$.

An expression of equality, such as $3 + 2 = n$, or $n - 2 = 3$, is called an *equation*.

The two sides of an equation are called the *members* of the equation. In the equation $n - 2 = 3$, $n - 2$ is the *first member*, and 3 is the *second member*.

The quantities which make up the members of an equation are called the *terms* of the equation. In the equation $n - 2 = 3$, n and 2 are the terms of the first member, and 3 is the term of the second member.

Writing problems as equations and then solving the equations is often the shortest and best way of working the problems. In stating a problem as an equation, the **unknown quantity,** or number to be found, is represented by a letter, as in the equation $n - 2 = 3$. Finding the value of the letter which stands for the unknown quantity is **solving** the equation.

Let us take a very simple problem, write it as an equation, and then solve the equation.

Tom said, "I am thinking of a number. If you add 4 to it, the sum will be 16. What is the number?"

To write this problem as an equation, first choose a letter to stand for the number you have to find. If you let h stand for Tom's number, your equation will be $h + 4 = 16$.

To solve the equation, ask yourself, "What added to 4 makes 16?" You know that since $16 - 4 = 12$, $12 + 4 = 16$. Therefore, Tom's number must be 12. To make sure that your answer is correct, write 12 in the equation in place of the letter h, and test to see if the equation is correct. $12 + 4 = 16$. $16 = 16$. The equation is correct.

Substituting the value of a letter or letters in an equation in place of the letter or letters and seeing whether the resulting statement is correct is **checking** the equation. Always check an equation after you have solved it.

The unknown quantity may not always be in the left-hand member of an equation. In stating as an equation "48 is 8 times a certain number," the equation is $48 = 8x$.

Make an equation for each of the following; then solve and check your equations. Use different letters to stand for the unknown numbers.

1. Four more than a number equals 10.
2. Seven less than a number equals 15.
3. A certain number taken from 17 leaves 9.
4. The product of 5 and a number equals 60.
5. A number divided by 6 equals 12.
6. One fourth of a number equals 9.
7. 14 is the sum of 9 and a certain number.
8. 32 equals a number divided by 2.
9. A certain number equals 12 times 7.

Make an equation for each of the problems on the following page. First find what the problem asks for and let a letter represent that quantity. Tell, in each case, for what the letter stands. Find the other facts in the problem and write the equation. Remember that the unknown quantity may be in either member of the equation. Solve the equation by finding the value of the unknown quantity. Check your answer.

10. If we add $7\frac{1}{2}$ T. of coal to the coal in our bin, we will have 12 T. How much coal is in our bin?
($x + 7\frac{1}{2} = 12$; $x = ?$)

11. Fred has saved \$15, which is 3 times the amount Jack has saved. How much has Jack saved?
($15 = 3x$; $x = ?$)

12. After using 5 qt. of oil from a full can, a man had 15 qt. left. How many quarts of oil did the can hold when full?

13. A house and a lot together are valued at \$9600. If the value of the lot is \$2400, what is the value of the house?

14. A section of farm land was divided into 4 equal parts. If each part contained 160 A., how many acres were there in the section before it was split up?

15. A coat listed at \$20 less a discount sold for \$15.50. What was the discount?

16. Mrs. Putnam paid \$1.12 for a 4-lb. roast. What was the price per pound?

17. One third of the distance between two towns is 65 mi. What is the entire distance?

MAKING PROBLEMS FROM EQUATIONS

Make a simple number problem from each of the following equations. For example, the equation $n - 12 = 6$ could show the facts stated in this problem: After spending \$12 for a dress, Edna had \$6 left. How much money did she have at first?

1. $n + 6 = 18$ **3.** $4b = 28$ **5.** $23 = 35 - x$

2. $44 - h = 12$ **4.** $\frac{m}{5} = 16$ **6.** $\frac{t}{9} = 14$

The druggist in the picture is filling an order for 6 oz. of boric acid powder. First he put a 2-oz. and a 4-oz. weight on one side of the scales. Now he is adding powder to the other side until the weight of the powder just equals that of the 2 and 4-oz. weights.

If the customer were to change his mind and say he wanted only 4 oz. after the 6 oz. had been weighed, the druggist would take the 2-oz. weight from one side of the scales and 2 oz. of powder from the other so that the scales would balance again.

An equation is like a pair of scales. It is kept in balance by adding the same amount to both sides or by subtracting the same amount from both sides. Both sides can also be multiplied or divided by the same amount and the equation will still be in balance.

1. Add 4 to both sides of the equation $4 + 8 = 12$. $4 + 8 + 4 = 12 + 4$. Check to see if the first member still equals the second member.

2. Subtract 4 from each member of the equation $4 + 8 = 12$. $4 + 8 - 4 = 12 - 4$. Does the equation still balance?

3. Now multiply the terms of each member of the equation by 4. Are the members still equal?

$$\begin{array}{r} 4 + 8 = 12 \\ \times 4 \quad \times 4 \\ \hline 16 + 32 = 48 \end{array}$$

4. If you divide the terms of each member of the equation by 4, does one member still equal the other?

$$\begin{array}{r} 1 + 2 = 3 \\ \hline 4)4 + 8 = 12 \end{array}$$

Here is another method by which we can find the unknown quantity in an equation.

In the equation $x - 8 = 15$, you can see that in the first member 8 has been taken from x. If you add 8 to $x - 8$, you put back what has been taken away and have only x, the unknown quantity, again. But in order to keep the equation balanced, you must also add 8 to the second member.

How to Solve an Equation by Addition

Solve the equation $x - 8 = 15$.

$$x - 8 = 15$$
$$x - 8 + 8 = 15 + 8$$
$$x = 23 \text{ Ans.}$$

Check: $23 - 8 = 15$
$15 = 15$

To solve an equation by addition, add to both members the number which will leave only the unknown quantity in one member.

Check your answer.

Rule 1. If the same number is added to both members of an equation, the members remain equal.

Solve these equations by addition, and check your answers:

1. $c - 2 = 2$ **5.** $94 = y - 20$ **9.** $y - \frac{3}{4} = 8$

2. $x - 7 = 3$ **6.** $r - 87 = 105$ **10.** $5\frac{1}{2} = m - 12\frac{1}{2}$

3. $m - 4 = 6$ **7.** $200 = y - 150$ **11.** $r - 7 = 2.5$

4. $b - 12 = 30$ **8.** $d - 40 = 90$ **12.** $13.6 = s - 1.25$

13. After selling $6\frac{1}{2}$ A. of his woodlot, Mr. Adams had $12\frac{1}{2}$ A. left. How many acres did he have at first?

14. Joe bought a camera for \$2.25 less than the regular price. If he paid \$8.50, what was the regular price?

In the first member of the equation $b + 3 = 39$, you can see that 3 has been added to b. If you subtract 3 from $b + 3$, you take away what was added and have only b left. To keep the equation balanced, you must also subtract 3 from the second member.

How to Solve an Equation by Subtraction

Solve the equation $b + 3 = 39$.

$b + 3 = 39$

$b + 3 - 3 = 39 - 3$

$b = 36$ Ans.

Check: $36 + 3 = 39$

$39 = 39$

To solve an equation by subtraction, subtract from both members the number which will leave only the unknown quantity in one member.

Check your answer.

Rule 2: If the same number is subtracted from both members of an equation, the members remain equal.

Solve these equations by subtraction, and check your answers:

1. $x + 3 = 15$
2. $4 + m = 21$
3. $24 = x + 6$
4. $n + \frac{1}{2} = 7$
5. $62\frac{1}{2} + d = 83\frac{1}{4}$
6. $7\frac{1}{8} = s + 4\frac{1}{2}$
7. $c + .4 = 2.2$
8. $3.8 + t = 7.1$
9. $78.5 = r + 54.9$

10. If I add \$5 to my savings account, I will have \$63. How much have I now in my account?

11. Goods which had cost \$36 were sold at a profit for \$41. How much was the profit?

12. The distance between two towns by route A is 81 mi., or 15 mi. more than by route B. What is the distance by route B?

In the equation $\frac{c}{8} = 16$, notice that c, the unknown quantity, is the numerator of a fraction. You can see that in the first member c has been divided by 8. If you multiply $\frac{c}{8}$ by 8, you will have $\frac{8c}{8}$, or c. To keep the equation balanced, you must also multiply the second member by 8.

How to Solve an Equation by Multiplication

Solve the equation $\frac{c}{8} = 16$.

$\frac{c}{8} = 16$

$\frac{c}{\cancel{8}} \times \cancel{8} = 16 \times 8$

$c = 128$ Ans.

Check: $\frac{128}{8} = 16$

$16 = 16$

To solve an equation by multiplication when the unknown quantity is the numerator of a fraction, multiply both members by the number which will leave only the unknown quantity in one member. Check your answer.

Rule 3. If both members of an equation are multiplied by the same number, the members remain equal.

Solve these equations by multiplication, and check your answers:

1. $\frac{m}{6} = 3$
2. $\frac{y}{8} = 4$
3. $25 = \frac{a}{4}$
4. $\frac{1}{12}r = 8$
5. $16 = \frac{x}{3}$
6. $2 = \frac{1}{5}n$
7. $\frac{h}{4} = 9\frac{1}{2}$
8. $\frac{r}{81} = 5\frac{1}{4}$
9. $\frac{t}{17} = 3\frac{1}{5}$
10. $\frac{b}{8} = 9.6$
11. $\frac{c}{17} = 4.27$
12. $\frac{s}{49} = .677$

13. If $\frac{1}{5}$ of a bushel of wheat weighs 12 lb., how much does a bushel of wheat weigh?

14. Jim has read 41 pages of his new book. This is $\frac{1}{8}$ of the pages in the book. How many pages are there in Jim's book?

In the equation $6x = 168$, you can see that in the first member x has been multiplied by 6. If you divide the first member by 6, you have only x, the unknown quantity, left. But in order to keep the equation balanced, you must also divide the second member by 6.

How to Solve an Equation by Division

Solve the equation $6x = 168$.

$6x = 168$

$\frac{6x}{6} = \frac{168}{6}$

$x = 28$ Ans.

Check: $6 \times 28 = 168$

$168 = 168$

To solve an equation by division, divide both members by the number which will leave only the unknown quantity in one member.

Check your answer.

Rule 4: If both members of an equation are divided by the same number, the members remain equal.

Solve these equations by division, and check your answers:

1. $4y = 48$ **3.** $24 = 8m$ **5.** $4b = 1.8$ **7.** $2.5x = 150$

2. $9c = 72$ **4.** $3n = 21$ **6.** $.96 = 12c$ **8.** $8.4 = 1.2d$

9. Five times the distance between two towns equals 20 miles. How far apart are the towns?

10. Mat's father is 36 years old. His age is 4 times Mat's age. How old is Mat?

PRACTICE IN SOLVING EQUATIONS

State which rule you would use to solve each of these equations. Then solve and check each equation.

1. $x - 5 = 11$ **3.** $q + 22\frac{1}{2} = 51\frac{3}{4}$ **5.** $68 = 17c$

2. $5m = 65$ **4.** $\frac{n}{6} = 14$ **6.** $\frac{x}{8} = 4.8$

You have learned that you can add things which are alike, as 50 trees and 12 trees, or \$90 and \$80. You can also subtract things which are alike. In the same way, you can add or subtract **like terms** in algebra; that is, terms which contain the same letter, as $5a + 7a + 4a = 16a$ and $76n - 43n = 33n$. This is called **collecting terms.**

Collect the terms in each of these examples:

1. $17y + 6y + 5y$
2. $2x + 5x + 7x + 4x$
3. $35b - 18b$
4. $50n - 26n$

You can see that 6 ft. − 3 ft. + 1 ft. = 4 ft. In the same way, $6n - 3n + n = 4n$. Notice that $1n$ is written as n.

Collect the terms in each of these examples:

5. $4x - 3x + 2x$
6. $7b + 9b - 12b$
7. $17k - 14k + 20k - k$
8. $14r + r - 6r - 8r$

Take this problem: Four times a number plus 2 times the number plus 3 times the number = 18. Find the number. Let m stand for the number. Then the equation for this problem is $4m + 2m + 3m = 18$. Collecting the terms of the first member of the equation, we have $9m = 18$. Then $m = 2$.

Solve these equations and check your answers:

9. $2y + 8y + 5y = 60$
10. $5x + 9x + 4x = 54$
11. $7m + 6m + 8m = 105$
12. $24b - 18b = 72$
13. $126a - 84a = 63$
14. $2c + 3\frac{1}{2}c + 1\frac{1}{2}c = 35$
15. $15n - 7n + 3n = 121$
16. $12x - 9x + 4x - x = 126$
17. $8d - 2\frac{1}{4}d + 1\frac{1}{4}d = 77$
18. $\frac{3}{4}x + 1\frac{1}{2}x - \frac{1}{4}x = 14$

19. The perimeter of a triangle is 40″. If one side is $2\frac{1}{4}$ times as long as the base and the other side is $1\frac{3}{4}$ times as long as the base, how long is the base?

To solve an equation like $7y - 8 = 34$, you need to use more than one rule in order to get only the unknown quantity in one member.

How to Solve an Equation, Using More Than One Rule

Solve the equation $7y - 8 = 34$.

$$7y - 8 = 34$$
$$7y - 8 + 8 = 34 + 8$$
$$7y = 42$$
$$y = 6 \text{ Ans.}$$

Check: $7 \times 6 - 8 = 34$

$34 = 34$

Use the proper rules until you have only the unknown quantity in one member of the equation.

Check your answer.

What two rules were used in finding the value of y?

Study each of the following equations, and tell which rules you would use in solving it. Then solve the equation and check your answer.

1. $2d + 4 = 16$
2. $5m - 2 = 13$
3. $17 + 4p = 53$
4. $81 = 16t + 9$
5. $\frac{1}{6}c + 2 = 4$
6. $\frac{1}{4}r - 3 = 6$
7. $10 = \frac{1}{2}t - 7$
8. $\frac{1}{8}n + 10 = 42$
9. $16 = 6p + 4.6$
10. $3 + .7a = 17$
11. $6y + .06 = 4.2$
12. $2x - 5.6 = .04$

13. $\frac{3}{4}c = 24$. To solve this equation, first write $\frac{3}{4}c$ as $\frac{3c}{4}$. Then $\frac{3c}{4} = 24$. What is the value of c?

14. $\frac{2}{3}x = 14$
15. $\frac{4}{5}m = 18$
16. $21 = \frac{7}{9}t$
17. $\frac{7}{8}r = 5.6$

18. $b + \frac{1}{4}b = 10$. Collecting terms in the first member, we have $b + \frac{1}{4}b = 1\frac{1}{4}b$, or $\frac{5}{4}b$. Writing $\frac{5}{4}b$ as $\frac{5b}{4}$, we have $\frac{5b}{4} = 10$. What is the value of b?

19. $x + \frac{1}{2}x = 15$
20. $24 = m - \frac{1}{5}m$
21. $3t + \frac{1}{4}t = 39$

You will recall that when two numbers are compared by division, the result is called their **ratio**; as $\frac{4}{2}$ is the ratio of 4 to 2.

Remember that only like things can be compared. For example, we can compare dollars with dollars and feet with feet. We cannot compare dollars with feet or feet with yards.

Write as ratios: 2 min. to 10 min.; \$8 to \$2; 2 ft. to 4 ft.

The ratio $\frac{6}{3}$ equals 2. The ratio $\frac{10}{5}$ also equals 2. As these ratios have the same value, they may be written as the equation $\frac{6}{3} = \frac{10}{5}$.

An equation which expresses equality between two ratios is a *proportion*. $\frac{6}{3} = \frac{10}{5}$ is a proportion.

Since there are 5 nickels in 25¢, there must be 10 nickels in 50¢. If this statement is true, the ratio of nickels to nickels ($\frac{5}{10}$) should equal the ratio of cents to cents ($\frac{25}{50}$), and this proportion should be true: $\frac{5}{10} = \frac{25}{50}$. To test this proportion, reduce both ratios to lowest terms. $\frac{5}{10} = \frac{1}{2}$. $\frac{25}{50} = \frac{1}{2}$. $\frac{1}{2} = \frac{1}{2}$. Since the two ratios are equal, the proportion is true, and so is the statement from which the proportion was made.

Make and test a proportion for each of the following:

1. If a man can drive 10 mi. in 15 min., he can drive 40 mi. in an hour.

2. Tom read 300 words in 2 minutes. At that rate, he could read 1500 words in 10 minutes.

3. If 2″ in a scale drawing stand for 40′, 9″ in the drawing stand for 180′.

Many a problem may be solved by setting it up as a proportion and solving the proportion.

How to Solve a Proportion

What length on a map represents 400 mi. if 2″ on the map represents 160 mi.?

$\frac{x}{2}$ = ratio of map lengths

$\frac{400}{160}$ = ratio of distances

Let x stand for the unknown quantity.

From the facts in the problem, write two equal ratios, and set them up as a proportion.

$$\frac{x}{2} = \frac{400}{160}$$

$$\frac{x}{2} \times 2 = \frac{\overset{5}{\cancel{400}}}{\underset{2}{\cancel{160}}} \times \cancel{2}$$

To solve the proportion, leaving only x on one side, multiply both sides by the denominator of the fraction containing x.

$$x = 5$$

Check your answer.

Check: $\frac{5}{2} = \frac{400}{160}$ $\quad 2\frac{1}{2} = 2\frac{1}{2}$

5″ on the map represents 400 mi.

Find the unknown quantity in each of the following proportions. Check your answer.

1. $\frac{x}{8} = \frac{24}{12}$
2. $\frac{x}{27} = \frac{4}{9}$
3. $\frac{x}{15} = \frac{90}{5}$
4. $\frac{5}{6} = \frac{x}{24}$
5. $\frac{x}{4} = \frac{16}{32}$
6. $\frac{x}{9} = \frac{36}{18}$
7. $\frac{c}{18} = \frac{4}{6}$
8. $\frac{48}{16} = \frac{x}{9}$
9. $\frac{15}{18} = \frac{m}{540}$
10. $\frac{a}{300} = \frac{4}{200}$
11. $\frac{y}{16} = \frac{28}{112}$
12. $\frac{7}{15} = \frac{x}{45}$

13. How many acres can a farmer plow in 20 hr. if he plows at the rate of 6 A. in 8 hr.?

1. How much can John save in 10 wk. if he saves at the rate of \$3 every 2 wk.?

2. How many cups of sugar will be needed to make a cake for 25 people if the recipe calls for 2 cups of sugar for a cake for 10 people?

3. How far will Mr. Harris travel in 16 da. if he travels at the rate of 930 mi. in 6 da.?

4. How many pounds of milk must a dairy farmer sell to receive \$42, if he receives \$2.80 for 100 lb. of milk?

5. Find the tax on property assessed at \$12,000 if the tax on property assessed at \$3500 is \$42.

6. What is the interest on a principal invested at 5% if the interest on the same principal at 6% is \$54?

7. If Tom can walk 6 mi. in 2 hr., how many miles can he walk in 5 hr. at the same rate?

This problem can be set up as the proportion $\frac{6}{x} = \frac{2}{5}$ or as the proportion $\frac{x}{6} = \frac{5}{2}$. Solve the proportion $\frac{x}{6} = \frac{5}{2}$.

In working the following problems by proportion, set up the proportions so that in each case the unknown quantity is the numerator of a fraction. Then solve the proportions.

8. A grocer's sales for 6 da. amounted to \$720. At that rate, what would his sales be for 26 da.?

9. Jane earns \$12.50 a week, and Dick earns \$15.50 a week. When Jane has earned \$75, how much will Dick have earned?

10. A snapshot is $2\frac{1}{2}$ in. long and $1\frac{1}{4}$ in. wide. An enlargement of it is $3\frac{3}{4}$ in. wide. How long is the enlargement?

The ratio of 5 to 4 is $\frac{5}{4}$, which may be written as 5:4. The proportion $\frac{5}{4} = \frac{15}{12}$ may also be written 5:4 = 15:12 or 5:4::15:12. The sign :: between two ratios stands for **equals.** This proportion is read **5 is to 4 as 15 is to 12.**

When a proportion is set up in the form 5:4 = 15:12 or 5:4::15:12, the first and fourth terms are called the **extremes,** and the second and third terms are called the **means.** In the above proportion, 5 and 12 are the extremes, and 4 and 15 are the means.

In any true proportion, the product of the means is equal to the product of the extremes.

Test to see if the following are true proportions:

1. 7:14 = 4:8 **2.** 6:8 = 9:12 **3.** 2:3::4:5

You can use the above rule to solve proportions.

How to Solve a Proportion

Solve the proportion x:20 = 24:30.

x:20 = 24:30	Find the product of the extremes and the product of the means.
$x \times 30 = 30x$ product of the extremes	To solve the proportion, write the two products as an equation, and solve the equation.
$20 \times 24 = 480$ product of the means	Check your answer.
$30x = 480$	
$x = 16$ Ans.	

Check: $16 \times 30 = 20 \times 24$ $480 = 480$

Solve and check the following proportions:

4. x:42 = 3:2 **6.** 7:6 = x:12 **8.** 14:18 = 21:x

5. 15:x = 5:4 **7.** 16:30 = x:45 **9.** 64:40::24:x

A formula, as you know, is a short way of stating a rule. Here are some formulas you have learned. Tell what each means.

Area of a rectangle: $A = lw$
Circumference of a circle: $C = 2\pi r$
Interest: $i = prt$

The great usefulness of formulas lies in the fact that they express statements that may be applied to many different cases. The formula $A = lw$, for example, expresses the area of any rectangle, no matter what its size or shape. The formula $C = 2\pi r$ expresses the circumference of any circle.

The same letter may appear in several formulas but may stand for different things. In the formula $C = 2\pi r$, r stands for radius. In the formula $i = prt$, r stands for rate. It is therefore necessary to know for what each letter in a formula stands.

1. What is the area of a rectangle $8'$ long and $4'$ wide? The formula is $A = lw$. Substituting 8 for l and 4 for w, we have $A = 8 \times 4$. Substituting known values for the letters in a formula is called **evaluating** the formula. Find the value of A. What is the area of the rectangle?

2. If the dimensions of the rectangle in problem 1 were doubled; that is, if the rectangle were $16'$ long and $8'$ wide, how many times would A be increased? Test to see.

3. If the width of the rectangle in problem 1 remained the same and the length were doubled, what would happen to the value of A? If l remained the same and w were doubled, what would happen to the value of A? Test to see.

You see that in using the formula $A = lw$, the value of A depends on any change in l or w. In the formula $C = 2\pi r$, the value of C changes if the value of r is changed. What is true in a change of values in the formulas for area and circumference is also true for changes in the values of letters in any formula.

USING FORMULAS

We can always find the value of any letter in a formula if we know the value of each of the other letters. Substituting the known values for the letters in the formula gives us an equation which can be solved for the unknown letter.

1. How much money must I invest at 4% to receive \$52 interest in 2 yr.? The interest formula is $i = prt$. Substituting in the formula the known values 52 for i, .04 for r, and 2 for t, we have this equation: $52 = p \times .04 \times 2$, or $52 = .08p$. Solve this equation for p, and check your answer.

2. At what rate of interest must I invest \$650 in order to receive \$52 interest in 2 yr.? Substituting in the formula $i = prt$ the known values 52 for i, 650 for p, and 2 for t gives this equation: $52 = 650 \times r \times 2$, or $52 = 1300r$. Solve the equation for r, and check your answer.

3. For how many years must I invest \$650 at 4% in order to receive \$52 interest? Substituting in the formula $i = prt$ the known values 52 for i, 650 for p, and .04 for r, we have the equation $52 = 650 \times .04 \times t$, or $52 = 26t$. Solve for t, and check your answer.

4. Make 3 problems similar to the above, and solve them by substituting the known values in the formula $i = prt$.

5. Using the formula $A = lw$, find the width of a rectangle having a length of 17 ft. and an area of 136 sq. ft.

1. How far can I drive in 5 hr. if I drive at the rate of 35 mi. per hour?

The formula for this problem is $d = rt$; d stands for distance, r for rate, and t for time. Substitute the known values in the formula, and find the value of d.

Suppose that in a problem the values of d and r are given and t has to be found. By using the formula $d = rt$ as an equation, you can make a formula for finding t.

To find t in the formula $d = rt$, divide both members of the equation by r.

$$d = rt$$
$$\frac{d}{r} = \frac{rt}{r}$$
$$\frac{d}{r} = t \text{ or } t = \frac{d}{r}$$

2. How long will it take me to drive 175 mi. if I drive at the rate of 35 mi. per hour? Write the formula, evaluate it, and find the value of t.

When the values of t and d are given and r is to be found, using the equation $d = rt$ you can make a formula for r.

$$d = rt$$
$$\frac{d}{t} = \frac{rt}{t}$$
$$\frac{d}{t} = r \text{ or } r = \frac{d}{t}$$

3. Mr. Simmons drove 175 mi. in 5 hr. What was his average rate of speed per hour? Write the formula, evaluate it, and find the value of r.

4. Change the formula $A = lw$ to a formula for finding l. Using the formula you have made, find the length of a rectangle whose width is 80 ft. and whose area is 9600 sq. ft.

5. Change the formula $i = prt$ to a formula for finding p; for finding r; for finding t. Using the formulas, find:

a. p when $i = \$63$, $r = 6\%$, and $t = 1\frac{1}{2}$ yr.
b. r when $p = \$1600$, $t = 6$ mo., and $i = \$40$.
c. t when $p = \$2500$, $i = \$112.50$, and $r = 4\frac{1}{2}\%$.

*Optional material.

6. $C = 2\pi r$. What is the formula for r? Using the formula for r, find the radius of a circle having a circumference of 264″.

$$C = 2\pi r$$
$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$
$$\frac{C}{2\pi} = r \text{ or } r = \frac{C}{2\pi}$$

7. What is 35% of 54? The formula for this problem is $p = br$, in which p stands for the **percentage,** or result obtained by taking a per cent of a number; b for the **base,** or number of which a per cent is taken; and r for the rate per cent. Substituting the known values for the letters in this formula gives $p = 54 \times .35$. What is the value of p?

8. From the formula $p = br$, make a formula for finding b; for finding r.

Using your formulas, solve problems 9 and 10.

9. If 8% of a number is 36, what is the number?

10. A basketball team played 12 games and lost 5. What per cent of games played were lost?

11. If paring knives cost 15¢ apiece, how much will 2 doz. knives cost? Make a formula for this problem, letting C stand for cost, n for the number of articles bought, and p for the price per article. Using the formula, find the value of C.

12. Change the formula $C = np$ to a formula for n; to a formula for p. Using the formulas you have made, find:

a. n when $C = \$34.50$ and $p = \$11.50$.

b. p when $C = \$155.76$ and $n = 12$.

13. Using P for perimeter and s for side, the formula for finding the perimeter of a square is $P = 4s$. Change this formula to a formula for finding s, and find the length of one side of a square whose perimeter is 22 in.

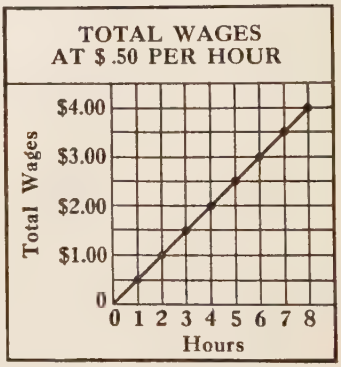
*Optional material.

Mr. Conrad does part-time work in a factory at the rate of \$.50 an hour. By using W for the total wage, r for the hourly rate, and t for the time in hours, we can express Mr. Conrad's total wage for any number of hours by the formula $W = rt$.

We can show the facts represented by the formula in a table as shown at the left below, from which we can read the total wage for any given number of hours.

TABLE OF WAGES AT \$.50 PER HOUR

Hours	Total Wage
1	\$ .50
2	1.00
3	1.50
4	2.00
5	2.50
6	3.00
7	3.50
8	4.00



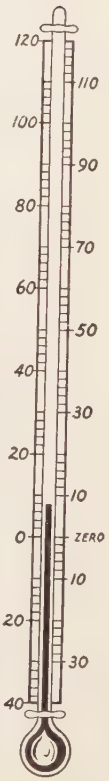
We can also make a graph of the facts represented in the formula $W = rt$, as shown above at the right. Suppose we let the vertical scale of the graph show the total wage for each number of hours from 1 to 8 hr., and let the horizontal scale show the hours.

1. Reading the graph, tell how much Mr. Conrad earned in 3 hr.; 7 hr.; 6 hr. How many hours had he worked if he earned \$1? \$2.50? \$4? \$3? Notice that when the time changes, the total wage also changes; and when the total wage changes, the time also changes.

2. A bus averages 30 mi. per hour. Using the formula $d = rt$, make a table and a graph to show the distances traveled for each number of hours from 1 to 10 hr.

*Optional material.

When Ben went to bed one winter night, the thermometer outside his window showed that the temperature was 8° above zero. In the morning, when he looked at the thermometer, the temperature was 3° below zero.



Look at the thermometer scale at the right. Notice that the degrees are numbered in opposite directions from 0°. **8° above zero** may be written **+8°**, and is read **plus 8°**. **3° below zero** may be written **−3°**, and is read **minus 3°**. Numbers written with a + sign before them are **positive numbers.** Numbers preceded by a − sign are **negative numbers.** Positive and negative numbers are used to represent numbers or quantities which are in opposite directions from some starting point.

Positive and negative numbers—also called **signed numbers**—are used in many ways. A height above sea level may be written with a + sign, as +500 ft.; and a depth below sea level, with a − sign, as −150 ft. A distance of 46° north of the Equator may be written +46°; and a distance of 70° south of the Equator, as −70°. In newspapers, a rise in the price of a stock or bond is shown by a +; and a fall, by a −.

Numbers that show opposites are often indicated in other ways than by + and − signs. Gains in business are usually written in black ink; and losses, in red ink. The letters B.C. and A.D. are used to show time before and after the coming of Christ. Tell what these mean: 1940 A.D.; 376 B.C.

1. Bring to class all the illustrations you can find which show uses of signed numbers and different ways of indicating numbers which show opposites.

$$\begin{array}{rr} +3 & -2 \\ +6 & -3 \\ \hline +9 & -5 \end{array}$$

Carol and Ralph were playing a game. Gains counted as + and losses as −. Carol's scores were +6 and +3. Her total score was therefore the sum of +6 and +3, or +9. Ralph's scores were −2 and −3. His total score, therefore, was the sum of −2 and −3, or −5.

To add two or more numbers having like signs, add their numerical values and prefix to the sum their common sign.

$$\begin{array}{rr} +6 & +2 \\ -3 & -3 \\ \hline +3 & -1 \end{array}$$

Suppose Carol's scores had been +6 and −3. Her total score would then have been the sum of +6 and −3, which you can see would give her a total score of +3. If Ralph's scores had been +2 and −3, his total score would have been the sum of +2 and −3, or −1.

To add two numbers having unlike signs, find the difference between their numerical values and prefix the sign of the larger number.

How to Add Several Numbers Having Unlike Signs

At a sale, a dealer gained \$3 on one article, lost \$2 on a second, gained \$1 on a third, and lost \$3 on a fourth. What was his net gain or loss?

$$\begin{array}{rr} +3 & -2 \\ +1 & -3 \\ \hline +4 & -5 \\ +4 & \\ -5 & \\ \hline -1 & \end{array}$$

Show each gain by a positive number, and each loss by a negative number.

Find the sum of the positive numbers.

Find the sum of the negative numbers.

Add the two sums.

The dealer had a net loss of \$1.

Add: 1. $\begin{array}{r} +6 \\ +9 \end{array}$ 2. $\begin{array}{r} -12 \\ -8 \end{array}$ 3. $\begin{array}{r} +16 \\ -8 \end{array}$ 4. $\begin{array}{r} -21 \\ +17 \end{array}$ 5. $\begin{array}{r} +3 \\ -8 \\ +9 \end{array}$ 6. $\begin{array}{r} +11 \\ -12 \\ -15 \end{array}$

*Optional material.

During 4 days last winter, Ted made temperature readings at two different times each day. To find the rise or fall in temperature, he subtracted the first reading from the second each day. He knew that if the temperature rose, the answer should be +; and if the temperature fell, the answer should be −. Here is the way he wrote the readings and his answers:

	1st Day	2nd Day	3rd Day	4th Day
Second Readings:	+9°	+7°	− 3°	− 2°
First Readings:	+3°	−2°	+12°	−12°
Rise or Fall:	+6°	+9°	−15°	+10°

On the thermometer scale on page 577, check Ted's answers by counting the number of degrees between the first and second readings for each day.

Now check Ted's answers in this way: In each example, think of the sign of the subtrahend as changed to the opposite sign, and add the numbers.

How to Subtract Signed Numbers

Find the difference in height between a place 168′ above sea level and a point 130′ below sea level.

Write the numbers as signed numbers.

+168
−130
+298

To subtract the signed numbers, think of the sign of the subtrahend as changed to the opposite sign, and add the numbers.

The difference in height is 298′.

Subtract: **1.** +56 / −18 **2.** +42 / +16 **3.** −54 / +16 **4.** −21 / −26 **5.** −42 / −64 **6.** +16 / −78

*Optional material.

Work the examples in column A. If you fail on any example, do the examples opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Drill	C Review
1. Making and Solving Equations		
57 is the sum of a number and 21.	9 less than a number equals 4.	558-559
23 equals one third of a number.	7 times a number equals 63.	
A number divided by 12 equals 9.	16 equals a number divided by 6.	
2. Solving Equations by Rules		
$a - 16 = 44$	$x - 15 = 40$ $\quad y - 12 = 28$	562
$x + .4 = 1.5$	$b + .8 = 3$ $\quad d + \frac{3}{4} = \frac{3}{2}$	563
$\frac{n}{3} = 17$	$\frac{r}{8} = 5$ $\quad \frac{t}{7} = 11$	564
$7d = 49$	$9c = 45$ $\quad 6q = 36$	565
$2a + 7a - 5a = 72$	$8m - 6m + 3m = 105$	566
$5x - 4 = 16$	$4y + 8 = 32$ $\quad 8m - 14 = 18$	567
$\frac{3}{4}t = 36$	$\frac{2}{3}r = 22$ $\quad \frac{4}{5}m = 16$	
$x + \frac{1}{3}x = 24$	$n - \frac{2}{5}n = 27$ $\quad b + \frac{1}{4}b = 65$	
3. Solving Proportions		
$\frac{12}{6} = \frac{x}{8}$	$\frac{x}{5} = \frac{20}{4}$ $\quad \frac{8}{12} = \frac{x}{3}$ $\quad \frac{x}{70} = \frac{10}{50}$	569
$\frac{12}{x} = \frac{3}{4}$	$\frac{15}{10} = \frac{6}{x}$ $\quad \frac{9}{x} = \frac{18}{4}$ $\quad \frac{3}{x} = \frac{45}{15}$	570
$x : 15 = 16 : 30$	$x : 9 = 35 : 21$ $\quad 48 : 28 = 12 : x$	571
$8 : x = 32 : 16$	$7 : x = 49 : 14$ $\quad 30 : 18 = 15 : x$	
4. Using Formulas		
$(i = prt)$	$(C = 2\pi r)$	
Find r when $i = \$22$, $p = \$440$, and $t = 1$.	Find r when $C = 44$ and $\pi = \frac{22}{7}$.	573

Try to work these problems without using pencil or paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. If Ellen is 11 yr. old, express her age b years ago.

2. If one side of an equilateral triangle is s, express its perimeter.

3. Give an equation for the following: 5 more than a certain number equals 48.

4. Find the value of x in the equation $x + 4 = 8$.

5. Solve the equation $\frac{v}{12} = 5$.

6. Dick is thinking of a number. If he subtracts 12 from it, the remainder is 36. Give the equation, and find the number.

7. Mr. Andrews pays one third of a debt. He pays \$40. Give the equation, and find the amount of the debt.

8. Four times the width of a stream is 160 ft. Write the equation, and find the width of the stream.

9. Find the value of y in the equation $\frac{3y}{4} = 15$.

10. Solve the equation $2m + 3m + m = 9$.

11. Find the unknown quantity in this proportion: $\frac{x}{4} = \frac{4}{8}$.

12. If Joe earns \$6 in 3 da. how long will it take him to earn \$24 at the same rate? Solve by proportion.

***13.** In the formula $A = bh$, find the value of b if $A = 20$ and $h = 2$.

*Optional material.

1.
$508.67
62.39
135.46
79.85
222.97
545.83
83.09
876.54

2. Subtract:

5 da. 6 hr.
3 da. 19 hr.

3. $12\frac{7}{8} + 3\frac{2}{3} = ?$

4. $68 - 9\frac{3}{8} = ?$

5. $4\frac{3}{5} \times 6\frac{7}{8} = ?$

6. $16\frac{2}{3} \div 1\frac{1}{3} = ?$

7. Express $12\frac{7}{8}$ as a mixed decimal.

8. Divide 42,837 by $33\frac{1}{3}$.

9. Solve and check this equation: $4a + \frac{3}{4}a = 57$.

10. Solve this proportion: $\frac{18}{x} = \frac{12}{30}$.

11. A man had d dollars. After paying a bill of $15, he had $6 left. How many dollars did he have at first? Select the correct equation for this problem from the following, and solve and check the equation: $d + 15 = 6$; $d - 15 = 6$; $\frac{d}{15} = 6$.

12. A number added to $\frac{3}{4}$ of itself gives 56. What is the number?

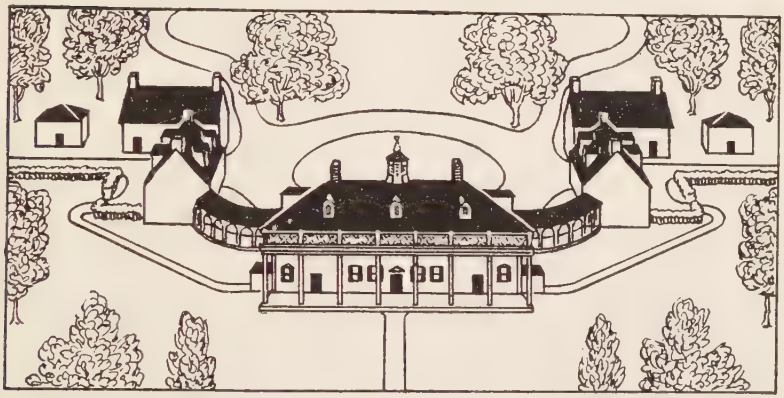
13. How many pounds of grass seed will a man need to sow 3000 sq. ft. of lawn at the rate of 5 lb. to 750 sq. ft.?

14. A rectangular lot has a perimeter of 180 yd. Its length is twice its width. What are the dimensions of the lot?

15. Alice wishes to divide a strip of cloth 48″ long into 2 pieces, one of which is to be 3 times as long as the other. How long should each piece be?

*Record your score on your Progress Chart. See page 321.

UNIT X. COMPARING AND USING GEOMETRIC FIGURES



The home of George Washington at Mount Vernon. A fine example of symmetry, or balance.

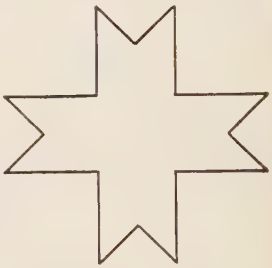
SYMMETRY, OR BALANCE

As you look at this picture of Mount Vernon, almost the first thing you notice is the way the two halves exactly balance. If you were to fold the picture through the middle from top to bottom, the buildings, walks, and shrubbery on one side would lie exactly over those on the other side. **Symmetry,** or balance, adds to the beauty of many designs.

When a plane figure, such as a rectangle, can be divided into two balanced halves, the line of division is called the **axis of symmetry,** and the figure is said to be **symmetrical with respect to a line.**

Solids which can be divided into two balanced halves are said to be **symmetrical with respect to a plane.** To illustrate: Cut a rectangular bar of soap into identical halves by slicing through the middle with a thin sheet of tin. The flat surface of the tin is a plane, and the soap on either side is symmetrical with respect to the plane.

1. Which of the drawings at the left are symmetrical? Which is not symmetrical? Could this object be drawn so as to appear symmetrical?



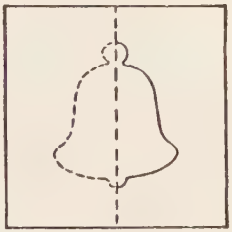
2. Trace the drawing at the right. Cut out your figure. See in how many different ways you can fold it into balanced halves. How many axes of symmetry do you find? Notice that all the axes cross at one point, called the **center of symmetry.**

3. Draw and cut out other symmetrical plane figures, and experiment to find how many axes of symmetry each has.

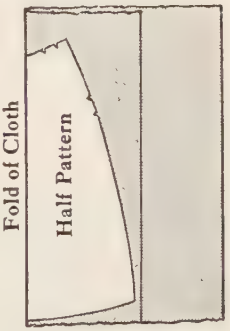
4. Draw a circle and cut it out. How many axes of symmetry can you find in it? Because you can find as many axes of symmetry as you desire in a circle, the circle is a perfect example of a symmetrical plane figure.

5. Could you divide a brick into two identical halves in more than one way? How many planes of symmetry does a rectangular solid have? A cube? A cone? A cylinder? A sphere? A sphere is said to be symmetrical with respect to its center. Can you tell why?

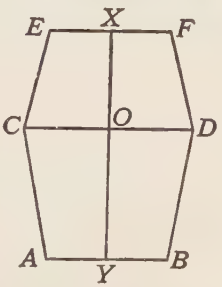
1. Fold a piece of paper vertically and crease it. Open the paper, and, using the crease as an axis line, draw in pencil the outline of one half of a bell. Trace the outline in ink, and, while the ink is still wet, fold the paper again along the axis and press the two parts together. Open the paper. Have you a symmetrical bell?



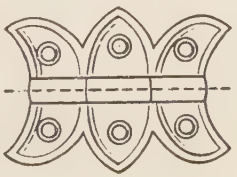
The parts of dress patterns are usually half patterns. The cloth to be cut is folded double, and the straight edge of the half pattern is laid along the fold as shown. When the material is cut out along the other edges of the pattern, the result is a symmetrical whole. Examine a dress pattern at home and see how it is made.

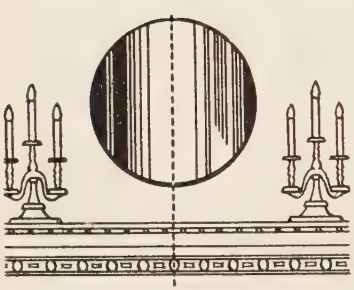


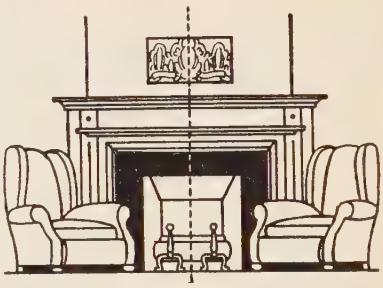
2. Draw the figure of the kite at the right. First draw the axis line XY, and mark points X, O, and Y where the cross-pieces of the kite cut the axis. At these three points, draw lines perpendicular to the axis line. Mark off the distance YA and the equal distance YB. Mark off the distances OC—OD and XE—XF in the same way. Connect points B, D, F and E, C, A by lines. Is the figure symmetrical?



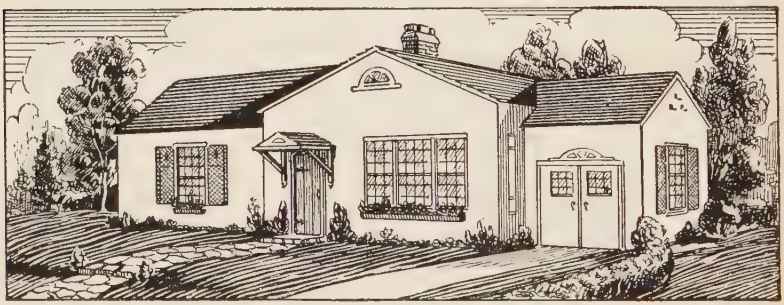
3. Test this figure for symmetry by measuring the distances either side of the axis line. Using ruler and compasses, draw the figure, making each measurement twice the length shown.







Pairs of figures are often placed symmetrically with respect to an axis of symmetry to give an attractive effect. How is this shown in the above drawings?



The above house is not symmetrical but gives an appearance of symmetry. The extension at the right somewhat balances the one at the left. The doorway in the center portion is somewhat balanced by the group of windows. This effect of symmetry is called **informal balance.**

1. Experiment with a small vase and a larger one on a mantel to give a symmetrical effect. Notice that if you place the smaller vase farther from the axis line than you do the larger one, you get a better effect of symmetry.

2. Bring to class pictures which show symmetrical arrangement and informal balance.

Examples of congruent, similar, and equivalent figures.

Figures which are exactly alike in size and shape are *congruent figures.*

1. Which of the above figures are congruent? Is a set of teaspoons congruent? Give other examples of congruent objects in your home.

Figures which are alike in shape but not in size are *similar figures.*

2. Which of the above figures are similar? Are garments which differ only in size similar? Give examples of similar figures in your home.

3. Are all rectangles similar? All triangles? All circles?

4. Draw a rectangle measuring $1'' \times \frac{3}{4}''$. Draw another rectangle measuring $2'' \times 1\frac{1}{2}''$. Are these two rectangles similar? What is the ratio of the length of the larger rectangle to the length of the smaller one? Of the width of the larger rectangle to the width of the smaller one? Are the two ratios equal?

In any two similar figures, the ratios of corresponding dimensions are equal.

Figures which are alike in size but not in shape are *equivalent figures.*

5. Which of the above figures are equivalent? Give other examples of equivalent figures.

Figure 1

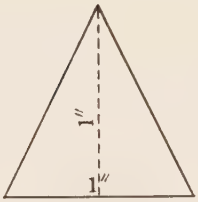


Figure 2

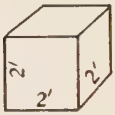


Figure 3

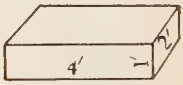


Figure 4

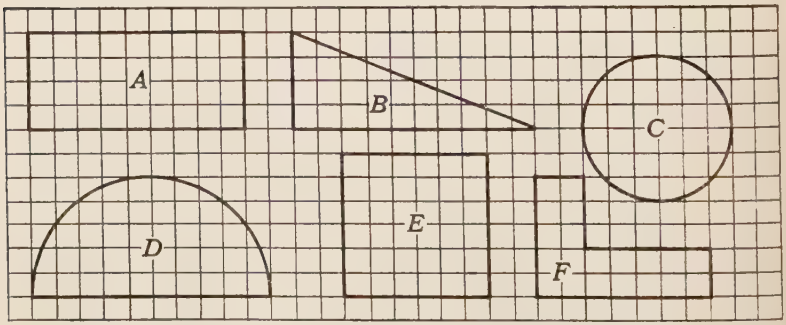
1. Find the areas of Figure 1 and Figure 2 above. What do you find?

2. Find the volumes of Figure 3 and Figure 4. What is true of their volumes?

Two plane figures are *equal* or *equivalent* when they have the same area. Two solids are equal or equivalent when they have the same volume.

3. Are a pint bottle and a pint measure equivalent figures? Why?

4. One triangle has a base of 2″ and an altitude of 3″. Another triangle has a base of 4″ and an altitude of 1″. Are the triangles equivalent? Explain.



5. Using the squared paper to help you find the areas, see if you can discover which of the figures shown above are equivalent.

The most common use of congruence is in the mass production of articles in standard sizes. Thousands of automobiles of one kind, for example, may be turned out quickly and cheaply because the parts used in one are identical in size and shape with those used in the others.

1. Name at least ten articles in which congruence is used.

2. Are most windows and doors of standard sizes? Why?

3. What buildings near your schoolhouse show balance or symmetry as you look at them from the street?

4. If a rectangle is drawn and inside it another rectangle is drawn each of whose sides is 1″ from the sides of the first rectangle, will the two rectangles be similar?

5. Bring to class symmetrical, congruent, and similar objects. Tell how the shape of each object adds to its usefulness, attractiveness, or convenience.

COMPARING FIGURES

1. How many times as much area has a lot 120 ft. by 240 ft. as a lot 60 ft. by 120 ft.?

2. A rectangular box having dimensions $1\frac{1}{2}$ times the dimensions of a similar box will hold how many times as much?

3. A cylindrical can twice as tall as another can and having the same diameter will contain how many times as much? A cylindrical can having the same height as another can and twice the diameter will contain how many times as much?

*4. How many times as much volume has a grapefruit 4″ in diameter as a grapefruit 2″ in diameter?

*5. Two spherical dips of ice cream each $1\frac{1}{2}$″ in diameter will contain what part of a single dip 3″ in diameter?

*Optional material.

1. Construct two triangles, each having a base of 4″, a left side of 3″, and a right side of $3\frac{1}{2}$″. Cut out the triangles and see if one fits exactly over the other as drawn. Are they congruent? Are the ratios between the corresponding sides of the triangles equal?

Two triangles are congruent if the three sides of one are equal to the corresponding three sides of the other.

2. At the left end of a line 3″ long, draw a 2″ line making an angle of 45° with the first line. Draw a third line to complete the triangle. Make a second triangle in the same way. Are the triangles congruent?

Two triangles are congruent if two sides and the included angle of one are equal to the corresponding two sides and included angle of the other.

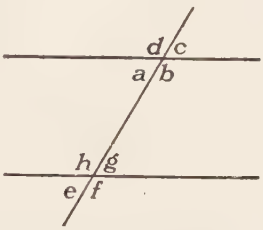
3. At the left end of a line 3″ long, draw a line making an angle of 40° with the first line. At the right end of the 3″ line, draw a line making an angle of 60° with the 3″ line. Extend the sides of the angles to form a triangle. Make a second triangle in the same way. Are the triangles congruent?

Two triangles are congruent if two angles and the included side of one are equal to the corresponding two angles and included side of the other.

4. Every triangle has three sides and three angles. From the directions given in exercises 1, 2, and 3, tell what measurements of a given triangle must be known in order to construct a triangle congruent to the given triangle. Is it necessary to know the lengths of all three sides and the sizes of the three angles?

5. Build other congruent triangles, using different measurements from those given in exercises 1, 2, and 3, and test to see if the three rules stated above always hold true.

Draw two parallel lines crossed by an oblique line, and mark the angles thus formed as shown in the figure at the right.



Angle a in the upper group corresponds in position to angle e in the lower group. The two angles are called **corresponding angles.**

1. Name the angle corresponding to $\angle b$; $\angle c$; $\angle d$.

2. Measure $\angle a$ and $\angle e$. Are they equal? Measure the other pairs of corresponding angles. What do you find?

When two parallel lines are cut by a third line, the corresponding angles are equal.

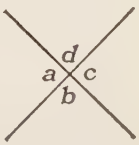
Show by measurement that $\angle a$ in the above figure equals $\angle g$. Angles placed like $\angle a$ and $\angle g$, between two parallel lines, are called **alternate interior angles.**

3. Name another pair of alternate interior angles in the above figure. Are they equal?

When two parallel lines are cut by a third line, the alternate interior angles are equal.

It is also true that if two lines are cut by a third line the two lines are parallel when (a) any two corresponding angles are equal or (b) any two alternate interior angles are equal.

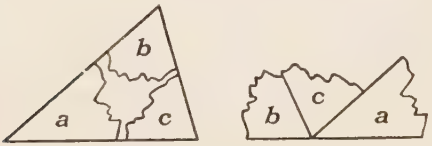
When two straight lines cross, the opposite angles thus formed are called **vertical angles.** In the figure at the right, $\angle a$ and $\angle c$ are vertical angles. $\angle b$ and $\angle d$ are also vertical angles.



4. Measure $\angle a$ and $\angle c$: $\angle b$ and $\angle d$. What do you find?

Vertical or opposite angles are always equal.

1. Draw a right triangle, an acute triangle, and an obtuse triangle. Measure the angles of each triangle and add them. Do you get the same sum for each of the three triangles? What is this sum?



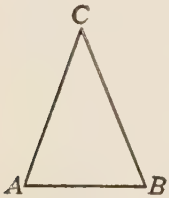
2. Draw a large triangle and mark the angles a, b, and c. Cut out the triangle, tear off the angles, and place them together in such a way that their vertices meet at one point, as in the figure above. What kind of angle is formed by the three angles? How many degrees has it?

The sum of the angles of any triangle is 180°.

3. If two angles of a triangle are 40° and 60°, what is the size of the third angle?

4. Find the number of degrees in the third angle of a triangle having two angles 30° and 55°; 75° and 40°; 115° and 20°; 87° and 49°; 90° and 45°; 60° and 60°.

5. Can a triangle have more than one right angle? More than one obtuse angle? A right angle and an obtuse angle? Any three given angles? Explain.



In the isosceles triangle at the left, side AC and side BC are equal sides. $\angle A$ is opposite side BC, and $\angle B$ is opposite side AC. Angles opposite the equal sides in an isosceles triangle are called **base angles.**

6. Can an isosceles triangle have each base angle equal to 90°? Can an isosceles triangle have an obtuse angle? Explain.

7. Measure $\angle A$ and $\angle B$ in the isosceles triangle. Are the angles opposite the equal sides equal?

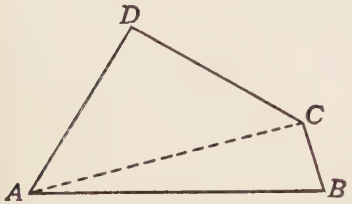
8. Draw other isosceles triangles and measure the base angles. Are they equal?

The base angles of any isosceles triangle are equal.

9. Draw an equilateral triangle and measure each angle. Are the angles equal? By testing, you will find that all equilateral triangles are equiangular triangles.

10. Draw several triangles of different shapes, and measure the angles and the sides of each. Do you find that in a triangle the longest side lies opposite the greatest angle?

QUADRILATERALS



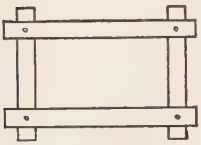
1. $ABCD$ is a **quadrilateral,** or four-sided figure. Since its sides are not equal, $ABCD$ is an **irregular quadrilateral.** Into what kind of figures does line AC divide $ABCD$? What is the sum of the angles in ABC? In ACD? What is the sum of the angles of the quadrilateral?

The sum of the angles of any quadrilateral is 360°.

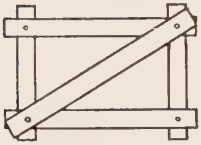
2. How many degrees are there in all the angles of a parallelogram? A rectangle? A square?

3. Is it possible for a parallelogram to have one right angle without having all angles right angles?

4. Draw on the blackboard a square, a rectangle, a parallelogram with slant sides, and an irregular quadrilateral. In what ways are all four figures alike? The first three? The first two? How is the square different from the other three figures?



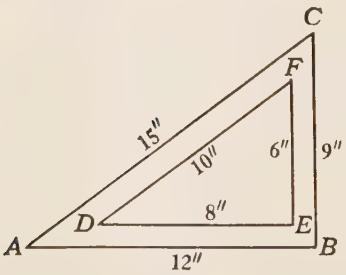
Make a triangle by nailing together three strips of wood. Use only one nail at each corner. Can you change the shape of the triangle by pushing on its sides? Is the triangle a rigid figure?

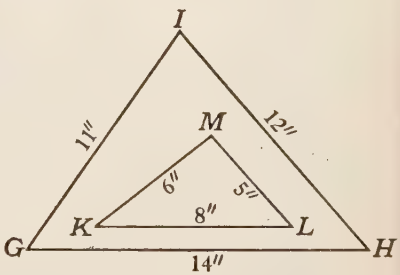


Make a rectangle by nailing together four strips of wood. Is the rectangle rigid? Now nail a fifth strip of wood diagonally across the rectangle. Into what kind of figures does the diagonal divide the rectangle? Is the rectangle now rigid? Why?

1. Tell in what ways builders use triangles to make structures strong and rigid.

SIMILAR TRIANGLES





1. Do triangles ABC and DEF have the same shape? Are they the same size? Are they similar triangles?

2. Are triangles GHI and KLM the same size? The same shape? Are they similar figures? Why?

3. In triangles ABC and DEF, do angles A and D appear to be equal? Angles B and E? Angles C and F? Measure the angles with a protractor. What do you find?

In similar triangles, the corresponding angles are equal.

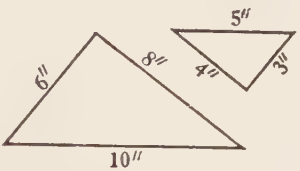
4. What is the ratio of side AB to side DE? Of side BC to side EF? Of side CA to side FD? Are all these ratios equal? Are these proportions true:

$$\frac{AB}{DE} = \frac{BC}{EF}, \quad \frac{AB}{DE} = \frac{AC}{DF}, \quad \frac{BC}{EF} = \frac{AC}{DF}?$$

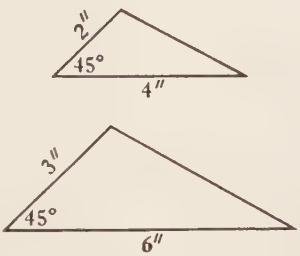
In similar triangles, the three pairs of corresponding sides have the same ratio and are proportional.

5. Are two triangles similar when two angles of one are equal to the corresponding two angles of the other? Why?

6. In what two ways can you find out if the two triangles at the right are similar? Do the positions of the two figures have anything to do with their similarity?



7. Construct one triangle having an angle of 45° enclosed by sides 2″ and 4″ long, and another triangle having an angle of 45° enclosed by sides 3″ and 6″ long. Do the corresponding sides of the two 45° angles have the same ratio? Are the triangles similar?



Two triangles are similar if an angle of one equals an angle of the other and the corresponding sides of the two equal angles have the same ratio.

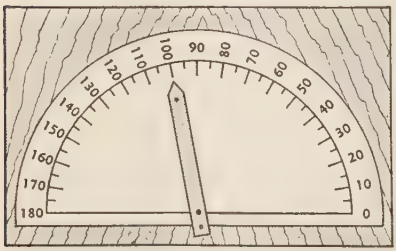
8. On squared paper, draw any triangle. Construct a similar triangle having sides twice as long. Compare the areas of the two triangles. If each side of a triangle is doubled, how many times is its area increased?

9. What measurements of a given triangle must be known in order to construct a triangle similar to the given triangle but not necessarily congruent to it?

Using a transit to measure angles.

Sometimes a distance cannot be measured directly because a body of water, a building, or some other object is in the way. However, such a distance can be found indirectly if it forms a side of a triangle other parts of which are known or can be measured.

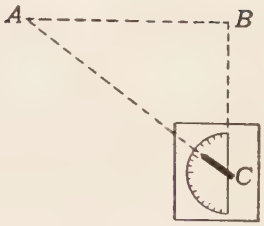
In finding distances by indirect measurement, a **transit** is used to measure angles. The above picture shows a surveyor using a transit. It consists mainly of a telescope which revolves above a protractor. The whole instrument is mounted on a tripod. Follow the directions below and make a homemade transit to use in measuring angles.



Glue a large protractor to a rectangular piece of board. At the center of the base line of the protractor, fasten a narrow pointer to the board by means of a screw so that the pointer can turn with the screw as a center. Drive a small nail into the pointer near each end to serve as sights. Your transit should look like the one above. Fasten it face upward to a camera tripod or to a stick that can be pushed into the ground.

To measure any angle, such as angle *BCA* in the diagram at the top of page 597, set your transit at point *C* so

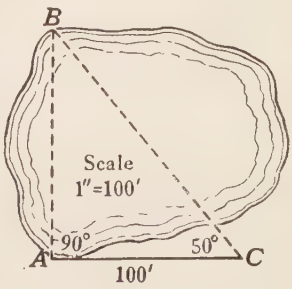
that 0° on the protractor scale is on line CB, and move the pointer until it is in line with A. Then read from the protractor the number of degrees in the angle BCA.



1. Find two objects, A and B, some distance apart out-of-doors. Using your transit, stand at some position C and measure $\angle BCA$.

To find the distance AB across a pond, Tom first marked points A and B (see diagram below). He then paced off a distance of 100′ at right angles to AB and marked point C. The three points A, B, and C formed the vertices of a triangle. Angle A, being a right angle, contained 90°. Using his transit, Tom found that $\angle C$ was 50°.

Tom next made a scale drawing of the outdoor triangle ABC, using the scale 1″ = 100′. First he drew the line AC. How long was it? With a protractor, he drew at A an angle of 90°, the same size as in the outdoor triangle. At C, he drew an angle of 50°, the same size as in the outdoor triangle. He then extended the sides of the angles until they met at B. The triangle in the scale drawing and the outdoor triangle were similar triangles because two pairs of corresponding angles were equal.



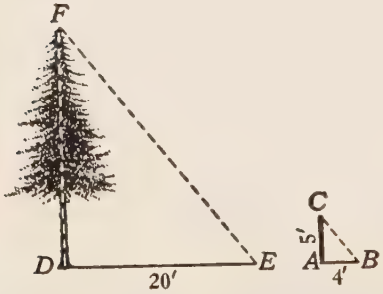
2. Find the actual length AB from Tom's drawing.

3. Jack found that two sides of a triangular park measured 120′ and 200′. Using his transit, he found the angle between the two sides to be 53°. Draw the triangle to a scale of 1″ = 40′, and find the length of the third side.

4. Find two trees on opposite sides of a building. Select a point from which you can see both trees. From this point, sight each tree with your transit and find the angle between the lines of sight. Measure the distance from the transit to each tree. Then make a diagram to scale and find the distance between the two trees.

5. Using transit and measuring tape, measure two angles and the included side of an outdoor triangle. Find by means of a scale drawing the lengths of the other two sides.

USING SIMILAR TRIANGLES TO FIND DISTANCES



Jack and Bob wanted to find the height of a tree. Not far from the tree stood a post 5′ tall. The drawing shows the tree and the post.

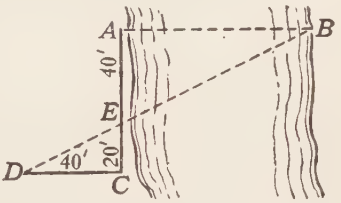
To find the height of the tree, the boys measured the lengths of the shadows cast at the same time by the tree and the post. Lines FE and CB show the direction of the sun's rays, which are parallel and strike the ground at equal angles E and B. The tree and the post both make right angles with the ground, so that angles D and A are equal. Triangles DEF and ABC are similar, since two angles of one are equal to the corresponding two angles of the other. Thus, the corresponding sides are proportional.

By measuring, the boys found that the shadow of the tree (DE) was 20 ft., and that the shadow of the post (AB) was 4 ft. The height of the post was 5 ft. Letting x stand for the height of the tree, they wrote this proportion from the equal ratios of the corresponding sides: $\frac{x}{5} = \frac{20}{4}$. What was the height of the tree?

1. A building casts a shadow 28 ft. long at the same time that a pole 6 ft. high casts a shadow 4 ft. long. How high is the building?

2. In two similar triangles ABC and DEF, side AC corresponds to side DF; and side AB, to side DE. If $AC = 2'$, $DF = 3'$, and $AB = 4'$, what is the length of DE?

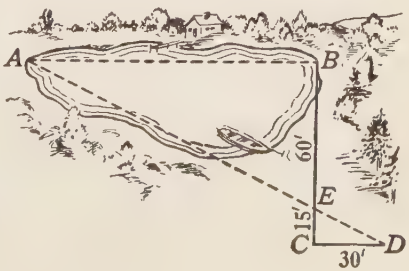
3. Some Boy Scouts wished to measure the width of a stream. They drove a stake at A directly opposite a tree B on the other bank. Then one Scout walked at right angles to AB from A to C and marked C with a stake. From C, he paced off a distance of 40 ft. at right angles to AC and marked point D. He then sighted from D to B, and another Scout marked point E where the line of sight DB crossed AC.



Angles A and C were equal because both were right angles. Angles CED and AEB were equal because they were vertical angles. Thus, triangles ABE and DCE were similar.

By measuring, the Scouts found that distance AE was 40 ft. and distance EC was 20 ft. Letting x stand for AB, the width of the stream, they wrote this proportion: $\frac{x}{40} = \frac{40}{20}$. What was the width of the stream?

4. To find the length, AB, of a pond, two boys found the measurements of BE, EC, and CD, as shown in the diagram at the right. What was the length of the pond?



Find:

1. 25% of 320
2. 8% of 1250
3. 27% of 3.5
4. $\frac{3}{8}$% of 160
5. $37\frac{1}{2}$% of 16
6. 375% of 16
7. 75% of \$420.80
8. 23% of 850 lb.
9. 4.5% of 680 ft.

Find what per cent the first number is of the second:

10. 35 of 50
11. 40 of 24
12. 21 of 6
13. 12 of 1800
14. 49 of 350
15. 137 of 400
16. 5.7 of 300
17. 265 of 250
18. 386 of 456

Find of what number:

19. 12 = 3%
20. 16 = 400%
21. 36 = $\frac{3}{4}$%
22. 720 = 120%
23. 49 = 7%
24. 136 = 34%
25. 287 = 4.1%
26. 186 = 31%
27. 4.8 = .5%

Find the interest on:

28. \$540 for 2 yr. 6 mo. at 6%; at 5%; at $4\frac{1}{2}$%.
29. \$876 at 5% for 9 mo. 6 da.; for 4 mo. 12 da.
30. \$1292 at 6% for 24 da.; at $3\frac{1}{2}$% for 5 mo.
31. \$342.50 for 7 mo. 15 da. at $4\frac{1}{2}$%; at 3%; at 7%.

Using the 60-day method, find the interest on:

32. \$458 for 69 da. at 6%; at 4%; at 7%.
33. \$3426 at 6% for 36 da.; for 18 da.; for 45 da.
34. \$452.50 at 5% for 54 da.; at 6% for 72 da.
35. \$829.60 for 75 da. at $4\frac{1}{2}$%; at 3%; at 5%.
36. \$2920 at 7% for 88 da.; for 96 da.; for 100 da.
37. \$3450 at 4% for 27 da.; at $5\frac{1}{2}$% for 48 da.

Numbers multiplied together to form a product are called the **factors** of the product. In the example $3 \times 2 = 6$, 3 and 2 are factors of 6. In the example $3 \times 3 = 9$, 3 and 3 are factors of 9. You remember that $3 \times 3 = 9$ may also be written $3^2 = 9$. The square of 3 is 9.

A *square* is the product of two equal factors.

Find the value of:

1. 9^2 3. 12^2 5. 24^2 7. 80^2 9. 45.2^2
2. 11^2 4. 15^2 6. 30^2 8. 75^2 10. 50.8^2

The *square root* of a number is one of its two equal factors.

Since $9 \times 9 = 81$, the square root of 81 is 9. The sign for square root is $\sqrt{\ }$. $\sqrt{81}$ means **the square root of 81.** $\sqrt{81} = 9$. Since $\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$, the square root of $\frac{9}{64}$ is $\frac{3}{8}$.

Read each of the following, and give its value:

11. $\sqrt{16}$ 13. $\sqrt{49}$ 15. $\sqrt{144}$ 17. $\sqrt{121}$ 19. $\sqrt{\frac{9}{25}}$
12. $\sqrt{25}$ 14. $\sqrt{36}$ 16. $\sqrt{100}$ 18. $\sqrt{\frac{4}{9}}$ 20. $\sqrt{\frac{25}{36}}$

Find the values of the following:

21. 18^2 23. 21.7^2 25. $(\frac{2}{3})^2$ 27. $\sqrt{\frac{9}{16}}$
22. 1.1^2 24. 4.05^2 26. $(\frac{4}{5})^2$ 28. $\sqrt{\frac{16}{25}}$

29. What are the dimensions of a square of linen containing 64 sq. in.?

30. A building is 280 ft. square. What is the area of the land covered by the building?

31. The perimeter of a square is 400 ft. What is the length of one side? What is the area of the square?

*Optional material.

Surveyors and others who work a great deal with squares and square roots save time by using tables like the one below.

TABLE OF SQUARES AND SQUARE ROOTS

Number	Square	Square Root	Number	Square	Square Root	Number	Square	Square Root
1	1	1.000	36	1296	6.000	71	5041	8.426
2	4	1.414	37	1369	6.083	72	5184	8.485
3	9	1.732	38	1444	6.164	73	5329	8.544
4	16	2.000	39	1521	6.245	74	5476	8.602
5	25	2.236	40	1600	6.325	75	5625	8.660
6	36	2.449	41	1681	6.403	76	5776	8.718
7	49	2.646	42	1764	6.481	77	5929	8.775
8	64	2.828	43	1849	6.557	78	6084	8.832
9	81	3.000	44	1936	6.633	79	6241	8.888
10	100	3.162	45	2025	6.708	80	6400	8.944
11	121	3.317	46	2116	6.782	81	6561	9.000
12	144	3.464	47	2209	6.856	82	6724	9.055
13	169	3.606	48	2304	6.928	83	6889	9.110
14	196	3.742	49	2401	7.000	84	7056	9.165
15	225	3.873	50	2500	7.071	85	7225	9.220
16	256	4.000	51	2601	7.141	86	7396	9.274
17	289	4.123	52	2704	7.211	87	7569	9.327
18	324	4.243	53	2809	7.280	88	7744	9.381
19	361	4.359	54	2916	7.348	89	7921	9.434
20	400	4.472	55	3025	7.416	90	8100	9.487
21	441	4.583	56	3136	7.483	91	8281	9.539
22	484	4.690	57	3249	7.550	92	8464	9.592
23	529	4.796	58	3364	7.616	93	8649	9.644
24	576	4.899	59	3481	7.681	94	8836	9.695
25	625	5.000	60	3600	7.746	95	9025	9.747
26	676	5.099	61	3721	7.810	96	9216	9.798
27	729	5.196	62	3844	7.874	97	9409	9.849
28	784	5.292	63	3969	7.937	98	9604	9.899
29	841	5.385	64	4096	8.000	99	9801	9.950
30	900	5.477	65	4225	8.062	100	10,000	10.000
31	961	5.568	66	4356	8.124	101	10,201	10.050
32	1024	5.657	67	4489	8.185	102	10,404	10.100
33	1089	5.745	68	4624	8.246	103	10,609	10.149
34	1156	5.831	69	4761	8.307	104	10,816	10.198
35	1225	5.916	70	4900	8.367	105	11,025	10.247

*Optional material.

To find the square of 23, first find 23 in the **Number** column. Opposite it, in the column headed **Square,** you find that the square of 23 is 529.

1. Find the square of 84; 61; 104; 54; 48.

To find the square of 3.6, first find 36 in the **Number** column. The square of 36 is 1296. The square of 3.6 is therefore 12.96. Why?

2. Find the square of 3.9; 8.7; .46; 10.2; 9.8.

To find the square root of 63, first find 63 in the **Number** column. Then read the number opposite it in the column headed **Square Root.** The square root of 63 is 7.937.

3. Find the square root of 29; 47; 105; 88; 72.

To find the square root of 3025, which is not in the **Number** column, look for the number in the column headed **Square.** Opposite it in the **Number** column is 55. Since 3025 is the square of 55, the square root of 3025 is 55.

4. Find the square root of 4225; 7056; 8464; 9604.

The **Square** column in the table opposite contains only **perfect squares,** like 1, 4, 9, 16, etc. We can, however, use the table to find the approximate square roots of numbers which are not perfect squares.

To find the approximate square root of 4516, find between what perfect squares 4516 lies. It lies between 4489 and 4624. The square root of 4489 is 67, and the square root of 4624 is 68. The approximate square root of 4516 is therefore between 67 and 68. 4516 is 27 more than 4489, and 4489 is 135 less than 4624. Therefore, the approximate square root of 4516 is $67\frac{27}{135}$, or 67.2.

5. Find to the nearest tenth: $\sqrt{267}$; $\sqrt{556}$; $\sqrt{1076}$.

*Optional material.

When a table of square roots is not available, you can find the approximate square root of a number by estimating and then testing.

How to Find the Approximate Square Root of a Number

Find the approximate square root of 20 to the nearest tenth.

20 is between 16 (4^2) and 25 (5^2). Therefore the square root of 20 is between 4 and 5.

Estimate a trial square root.

Since 20 is slightly nearer 16 than 25, try a root slightly nearer 4 than 5, as 4.4.

Square the trial root.

$4.4 \times 4.4 = 19.36$ (less than 20)

If the square of the trial root is too small, try a root one tenth larger.

$4.5 \times 4.5 = 20.25$ (more than 20)

$\sqrt{20} = 4.5$ approximately

If the square of the trial root is too large, try a root one tenth smaller.

Keep trying until you find the trial root whose square most nearly equals the given number. This gives the approximate square root of the number.

Find the approximate square roots to the nearest tenth:

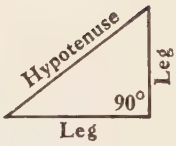
1. $\sqrt{30}$ 3. $\sqrt{125}$ 5. $\sqrt{220}$ 7. $\sqrt{252}$ 9. $\sqrt{3500}$
2. $\sqrt{50}$ 4. $\sqrt{180}$ 6. $\sqrt{178}$ 8. $\sqrt{215}$ 10. $\sqrt{3745}$

11. A square room contains 345 sq. ft. Find to the nearest tenth the length of each side of the room.

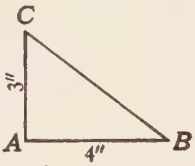
12. A surveyor laid out a park in the shape of a square. The park contained 1 A. Find the dimensions of the park to the nearest tenth of a rod.

*Optional material.

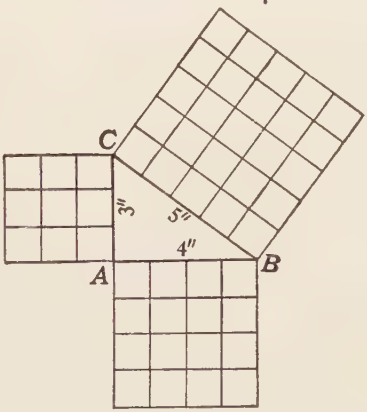
In a right triangle, the side opposite the right angle is called the **hypotenuse.** The two sides which form the right angle are called the **legs** of the triangle.



Draw on the blackboard two lines forming a right angle at A. From A, measure off 4 in. on one side to a point B, and 3 in. on the other side to a point C. Draw the line BC and measure it. BC should be 5 in. long.



On AB as a side, draw a square and divide it into 1-in. squares as shown in the drawing at the right. In the same way, draw a square on AC and one on BC, and divide these squares into 1-in. squares. How many square inches are there in the square on side AB? On side AC? How many square inches in the squares on both these sides? $16 + 9 = ?$ Is this sum the same as the number of square inches in the square on side BC? $16 + 9 = 25$, or $4^2 + 3^2 = 5^2$.



The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.

If we let c stand for the hypotenuse, and a and b for the other two sides, then:

Formula for the hypotenuse rule is:

$$c^2 = a^2 + b^2$$

*Optional material.

How to Find the Length of the Hypotenuse

A rectangular corner lot is 80′ by 60′. A diagonal path runs across it. Find the length of the path.

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Write the formula for the hypotenuse rule.

Since $c^2 = a^2 + b^2$, the formula for the hypotenuse (c) is

$$c = \sqrt{a^2 + b^2}.$$

$a = 80 \quad a^2 = 6400$

$b = 60 \quad b^2 = 3600$

$a^2 + b^2 = 10{,}000$

Square the lengths of the known sides (a and b), and substitute their sum in the formula for c.

$c = \sqrt{10{,}000} = 100$

Find the square root of the sum.

The length of the path is 100′.

From the lower figure on page 605, show that:

$$a^2 = c^2 - b^2 \text{ and } b^2 = c^2 - a^2.$$

The square of one leg of a right triangle is equal to the square of the hypotenuse minus the square of the other leg.

If $a^2 = c^2 - b^2$, $a = \sqrt{c^2 - b^2}$. If $c = 10$ and $b = 8$, $a = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$.

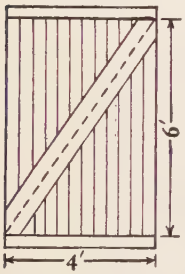
If $c = 10$ and $a = 6$, show that $b = \sqrt{10^2 - 6^2}$.

Find the unknown side in each of these right triangles:

	Hypotenuse	One Leg	Other Leg
1.	?	9″	12″
2.	?	16″	15″
3.	40″	?	32″
4.	25 yd.	20 yd.	?
5.	30 ft.	18 ft.	?

*Optional material.

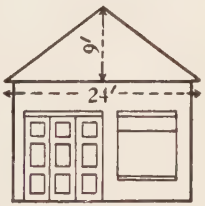
1. A rectangular sheet of cardboard is 24″ long and 18″ wide. Find the length of a diagonal of the sheet. As a check, make a drawing of the cardboard to scale and measure the diagonal.



2. To strengthen the door of his camp shack, Dick decided to nail a board diagonally across the door as shown in the drawing. How long a board does he need?

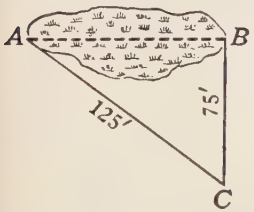
3. To brace a flagpole 35 ft. high, a wire is stretched from the top of the pole to the ground at a point 14 ft. from the foot of the pole. How long is the wire?

4. Find the length of the sloping side of the roof shown in the picture at the right.

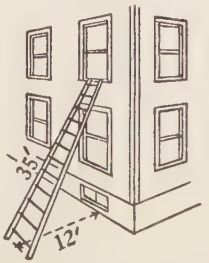


5. One side of an equilateral triangle is 16′ long. Find the height of the triangle.

6. To measure the length of a swamp on his father's farm, Jerry drove stakes A and B at each end as shown in the picture. He then laid off on the ground at right angles to AB a line BC, 75′ long. He measured the distance AC and found that it was 125′ long. What was the length of the swamp?



7. A painter placed a 35-ft. ladder to reach from the ground to a windowsill of a house as shown in the picture at the right. If the foot of the ladder is 12 ft. from the house, what is the height of the sill above the ground?



*Optional material.

Work the exercises in column A. If you fail on any exercise, do the exercise opposite it in column B. If you still make mistakes, review the pages given in column C.

A Test	B Test	C Review
1. Comparing Figures		
Are the following equivalent, congruent, or similar? Two rectangles, each 6″ long and 4″ high A 5″ square and a parallelogram having a base of 10″ and a height of $2\frac{1}{2}$″ A △ having sides 12″, 15″, and 18″ long and a △ having corresponding sides 8″, 10″, and 12″ long	Are the following equivalent, congruent, or similar? A 9″ square and a $\frac{3}{4}$′ square A 10″ square and a triangle having a base of 10″ and a height of 20″ A △ having sides 4″, 6″, and 8″ long and a △ having corresponding sides 6″, 9″, and 12″ long	587–588 594–595
2. Finding an Unknown Side of a Triangle		
Known sides of one △: 12′ and 8′; corresponding sides of a similar △: 9′ and ?′ Known sides of one △: 8′, 10′, and 12′; corresponding sides of a similar △: 4′, ?′, ?′	Known sides of one △: 16′ and 20′; corresponding sides of a similar △: 8′ and ?′ Known sides of one △: 9″, 12″, 15″; corresponding sides of similar △: 3″, ?″, ?″	598–599
3. Finding the Square Root and the Approximate Square Root*		
$\sqrt{169}; \sqrt{2500}; \sqrt{28}; \sqrt{70}$	$\sqrt{225}; \sqrt{4900}; \sqrt{32}; \sqrt{75}$	604
4. Using the Hypotenuse Rule*		
Using the formula $c^2 = a^2 + b^2$, find: c if $a = 15$ and $b = 20$ a if $c = 20$ and $b = 12$ b if $c = 60$ and $a = 36$	Using the formula $c^2 = a^2 + b^2$, find: c if $a = 12$ and $b = 16$ a if $c = 45$ and $b = 36$ b if $c = 40$ and $a = 32$	605–606

*Optional material.

Try to work these problems without using pencil or paper. Write only the answers. If you cannot do a problem mentally, work it on paper.

1. What is the square of 40?

2. Two angles of a triangle are 35° and 50°. How many degrees are there in the third angle?

3. A triangle has sides 4″, 7″, and 6″ long. Its largest angle is opposite what side?

4. An isosceles triangle has one base angle of 40°. How many degrees are there in each of the other angles of the triangle?

5. A triangle has a base of 6″ and a height of 5″. Another triangle has a base of 10″ and a height of 3″. Are the two triangles congruent or equivalent? Explain.

6. The base of a right triangle is 12′, and its height is 8′. If the base of a similar triangle is 9′, what is its height?

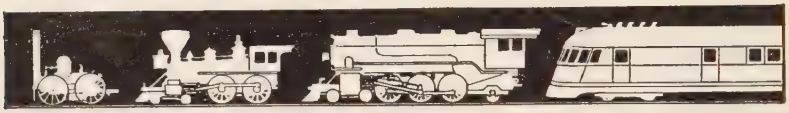
7. If a 4-ft. post casts a shadow 3 ft. long at the same time that a flagpole casts a shadow 15 ft. long, how high is the flagpole?

8. A 6-in. square is equivalent to a rectangle which has a base 9″ long. Find the width of the rectangle.

***9.** A square has an area of 121 sq. in. Find the length of a side of the square.

***10.** If the hypotenuse of a right triangle is 5 in. long and one leg is 4 in. long, how long is the other leg?

*Optional material.



1.
$ 96.90
154.67
500.36
94.70
878.97
377.00

2. $.0984 \times 7.05$

3. $3.06\overline{)7.65306}$

4. $66\frac{2}{3} \times \frac{3}{2} \times \frac{1}{8} = ?$

5. $3\frac{3}{7} \div 2\frac{2}{3} = ?$

6. 8% of $300 = \frac{1}{2}\%$ of ?

7. $37 = ?\%$ of 120

8. If two angles of a triangle are 64° and $37\frac{1}{2}°$, what is the size of the third angle?

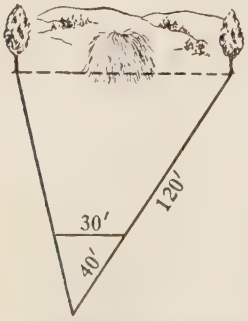
9. Find the square of 16.1.

10. Is a 24-in. square equivalent to a triangle which has a base of 36 in. and a height of 32 in.? Explain.

11. Compare the volumes of a 12″ cube and a 4″ cube.

12. Draw two rectangles, one 3″ by $1\frac{1}{2}$″, the other 6″ by 9″. Are they similar? Why?

13. A triangle has sides of 6′ and 4′ including an angle of 45°. Draw the triangle to the scale 1″ = 2′, and find the actual length of the third side.



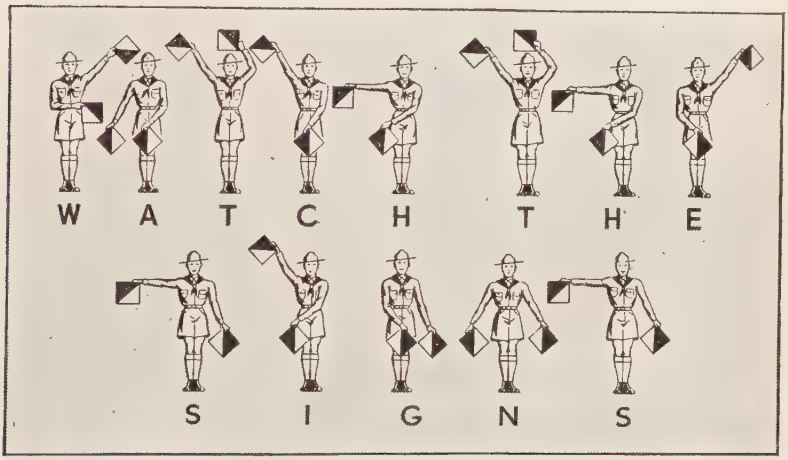
14. Two trees are on opposite sides of a haystack. From the drawing at the left, find the actual distance between the trees.

15. Construct a triangle having sides 4″, 5″, and 6″ long. Construct a similar triangle having a side 6″ long corresponding to the 4″ side of the first triangle.

*Record your score on your Progress Chart. See page 321.

UNIT XI. A FINAL CHECKUP

The tests in this unit make a very complete coverage of arithmetic and include the findings of a country-wide study of the questions asked in standard tests and those given by states, cities, counties, and other groups.



Watch the signs, and see how rapidly and accurately you can work these exercises mentally:

1.	$48 \div 12 = ?$	$48 + 37 = ?$	$48 - 29 = ?$	$48 \times 3 = ?$
2.	$48 \div 1000 = ?$	$48 - 8.3 = ?$	$48 \times .2 = ?$	$48 \div .4 = ?$
3.	$48 + 22\frac{1}{2} = ?$	$48 - 14\frac{1}{4} = ?$	$663\frac{3}{4} - 48 = ?$	$48 \times \frac{2}{3} = ?$
4.	$\frac{5}{6} \times 48 = ?$	$16\frac{2}{3} \times 48 = ?$	$48 \div \frac{1}{2} = ?$	$36 \div 48 = ?$

5.	25% of $48 = ?$	2% of $48 = ?$	$\frac{1}{2}$% of $48 = ?$
6.	$24 = ?$% of 48	$96 = ?$% of 48	$4.8 = ?$% of 48
7.	$48 = 4$% of ?	$48 = \frac{1}{2}$% of ?	$48 = 300$% of ?

8. Work the above exercises again, using each of these numbers in place of 48: 60; 72; 120; 600.

1. In a recent year, the gold mines of the United States produced 5,056,308 oz. of gold, valued at $176,970,800. Tell what each figure in these numbers stands for. Round the numbers to the nearest hundred thousand.

Copy examples 2 to 11 below. Keeping a record of your time, work the examples and check your answers.

2.
$$\begin{array}{r} \$2528.64 \\ 415.22 \\ 3975.51 \\ 416.43 \\ 5030.02 \\ 644.21 \\ \hline \end{array}$$

3. $236050 - 91879$

4. $8. - 1.6739$

5. 88×9600

6. 2117×3.1416

7. 9.038×40.7

8. $6\overline{)10 \text{ bu. } 2 \text{ pk.}}$

9. $31.2\overline{)28.08}$

10. $61\overline{)48832}$

11. $.073\overline{)193.448}$

12. If your working time for the above examples was more than 20 minutes, build and work a set of similar examples.

13. Change to decimals: $\frac{3}{8}$; $\frac{7}{5}$; $\frac{9}{40}$; $\frac{5}{16}$; $\frac{21}{50}$; $6\frac{19}{25}$.

14. Change to fractions or mixed numbers: .75; .64; .004; .625; 3.45; $.66\frac{2}{3}$; .875; 5.025.

15. Add $6\frac{2}{5}$, $18\frac{3}{4}$, $21\frac{1}{2}$, and $7\frac{1}{10}$.

16. Subtract $5\frac{11}{16}$ from $6\frac{3}{4}$.

17. $6\frac{3}{4} \times \frac{2}{3}$ is what part of $\frac{15}{2}$?

18. $\frac{3}{4} \times \frac{5}{8} \times 48 = ?$

19. Divide $6\frac{7}{8}$ by $3\frac{1}{3}$.

20. $\frac{3}{8} = \frac{1}{3}$ of ?

21. What is the ratio of $45 \times \frac{3}{5}$ to 75?

22. A man drove 240 mi. in 6 hr. 15 min. What was his average speed per hour?

23. A wagon loaded with wheat weighs 4125 lb. The wagon weighs 1125 lb. Find the value of the wheat at $1.10 per bu. (60 lb.)

Work examples mentally whenever possible.

1. 325% of 1200 = ? **2.** 24 = 8% of ? **3.** 600 = ?% of 300

4. $\frac{3}{4}$% of \$1600 = 50% of \$? **5.** \$840 less 25% = \$?

6. What per cent of this figure is shaded?

7. A farmer tested 129 kernels of seed corn. He found that only 43 grew. What per cent of the kernels grew?

8. If iron ore from a mine is $16\frac{2}{3}$% pure iron, how many tons of ore must be mined to get $3\frac{1}{2}$ T. of pure iron?

9. A bell weighing 875 lb. consists of 11% copper, 2% zinc, and the rest tin. How many pounds of each metal does the bell contain?

10. Twenty-five textbooks listed at \$1.40 with a discount of 20% were ordered for 25 pupils. Parcel-post charges were \$1.50. What was the cost per pupil?

11. By selling a horse for \$189, the owner lost 10%. At what price should he have sold the horse to gain 10%?

12. Cost, \$800; selling price, \$960; overhead, 10%. Find the rate of profit (a) based on the cost; (b) based on the selling price.

13. What is the difference between a direct discount of 35% and successive discounts of 20% and 15% on a bill of goods amounting to \$220?

14. Find the net proceeds of a sale of 160 baskets of peppers @ \$1.42. Commission, 5%; charges, 5¢ per basket.

15. Mr. Mott sells mattresses which cost him \$14.50 apiece at a price to allow 20% for overhead and 15% for profit. Find the selling price if the rates of overhead and profit are based (a) on the cost; (b) on the selling price.

Find the interest and the amount of the following:

	Prin.	Rate	Time		Prin.	Rate	Time
1.	\$900	5%	2 yr.	**4.**	\$1200	6%	1 yr. 3 mo.
2.	\$320	6%	30 da.	**5.**	\$648	6%	1 yr. 4 mo. 6 da.
3.	\$1564	4%	48 da.	**6.**	\$18.50	6%	89 da.

7. A year ago, Mr. Kenny borrowed \$5900 to buy a new home. The interest for a year was \$295. What rate of interest did he pay?

8. How much would you have to pay for a bank draft for \$968 if the fee is $\frac{1}{10}\%$?

9. A loan of \$1200 earns \$20 in 4 mo. What is the rate of interest?

10. Using the interest formula, find the interest on \$625 at 4% from June 1 to Sept. 2. Check your answer by using the 60-da. method.

11. Find the difference between the simple interest and the exact interest at 5% on \$588 for 48 days.

12. Find the proceeds of a 60-da. \$300 note without interest which is discounted at $5\frac{1}{2}\%$ on the day it is made.

13. A 90-da. trade acceptance for \$700, dated May 9, was discounted May 16 at 6%. Find the proceeds.

14. Find the difference between the simple interest at 4% and the interest at 4% compounded semiannually on \$420 for 1 yr. 6 mo.

15. Write a 60-da. note for \$560 at 6% interest, dated Aug. 7, in favor of J. S. Towles. Find the proceeds if the note is discounted Aug. 12 at 5%.

Find the fire insurance premium on the following for a term of (a) 1 yr.; (b) 3 yr.; (c) 5 yr. (3-yr. rate is $2\frac{1}{2}$ times 1-yr. rate; 5-yr. rate is 4 times 1-yr. rate.)

	Face	Yearly Rate per $100		Face	Yearly Rate per $100		Face	Yearly Rate %
1.	$2000	50¢	3.	$8250	$.68	5.	$8000	$\frac{3}{4}\%$
2.	$5000	40¢	4.	$9750	$.41	6.	$7350	$\frac{2}{5}\%$

7. Mr. Anderson insured property worth $12,000 for $\frac{2}{3}$ of its value at a yearly rate of 36¢ per $100. How much was his premium for 1 yr.?

8. On an automobile valued at $600, Mr. Royce paid for insurance as follows: Public liability, $36.35; property damage, $9.25; fire and theft, $1.05 per $100. How much did the total insurance cost him?

9. A shipment worth $6400 and insured at $\frac{7}{8}$ of its value at $2\frac{3}{4}\%$ was lost. Find the owner's loss, including premium.

10. Find the difference in the amount of annual premium for a $5000 ordinary life policy at $33.01 per $1000 and a $5000 20-payment life policy at $42.79 per $1000.

11. Mr. Claxton must give up his $4500 endowment policy. He can surrender it for cash at $294 per $1000, or take paid-up insurance of $393 per $1000. How much can Mr. Claxton get (a) in cash; (b) in paid-up insurance?

12. Mr. Ellis can buy an annuity yielding a quarterly income of $25, for $2561.10. He wishes a quarterly income of $350. Make and work a problem using these facts.

13. Mr. Wayne's accident policy costs $63.26 a year. When he broke an arm, the insurance company paid $75 for surgeon's fees and $25 a week for $5\frac{1}{2}$ weeks. How much more than one year's premium cost did Mr. Wayne receive in benefits?

Find the missing values:

	Valuation	Tax	Rate		Valuation	Tax	Rate
1.	$200,000	$3000	?%	5.	$6000	?	$1\frac{1}{4}$%
2.	$40,000	$1200	? mills per $1	6.	$4000	?	4 mills per $1
3.	$8000	$140	? per $100	7.	$8750	?	$2.18 per $100
4.	$90,000	$2812.50	? per $1000	8.	$19,425	?	$34.15 per $1000

Tell *how* to work each problem before working it.

9. Mr. Powell bought a theater ticket for $4.40, which included a 10% federal tax. How much tax did he pay?

10. Mr. Powell owns a store which is assessed at $\frac{3}{4}$ of its market value of $18,000, and goods and fixtures assessed at $6000. Find his tax at $3.25 per $100.

11. Last year, Mr. Powell used 6465 gal. of gasoline in his delivery car and his private car. The state tax was 4¢ per gallon; and the federal tax, 1¢. Find his total gasoline tax for the year.

12. Mr. Powell collected from his customers last year a sales tax of 2% on sales amounting to $157,386. How much tax did he collect?

13. When Mrs. Powell returned from a trip abroad, she brought home an oriental rug valued at $457. How much was the duty on it at 45% ad valorem?

14. Mr. Powell's salary last year was $3600, and he received $1264 from other sources. Find his normal federal income tax at 4% if he was allowed deductions of $560 for taxes, interest, and charity contributions; $2900 as a married man with one child; and a credit of 10% of his earned income.

1. The Conroy Company has a capital stock of \$80,000, par value \$50 a share. How many shares are there?

2. A company pays \$1200 in dividends on a capital stock of \$200,000. Find the rate of dividend and the dividend per \$100 share.

3. Each month for a year, Mr. Alden bought one \$50 United States Savings Bond for \$37.50. How much did he invest in these bonds in one year? What was the total face value of the bonds?

4. Mr. Barr invested \$5840 in a mortgage loan, at $5\frac{1}{2}\%$ payable quarterly. How much was each interest payment?

5. How much should Mr. Warner receive in dividends on 60 shares of stock, par value \$100 a share, if the dividend rate is 4%? If the dividend is \$2.50 a share?

Find the total cost of:

6. 200 shares of stock at 70, brokerage 20¢ a share.

7. 400 shares of stock at $96\frac{1}{4}$, brokerage 22¢ a share.

8. Six \$500 bonds at $88\frac{1}{2}$, brokerage \$1.25 per bond.

Find the proceeds of the sale of:

9. 300 shares of stock at 25, brokerage 15¢ a share.

10. 1200 shares of stock at $107\frac{3}{8}$, brokerage 23¢ a share.

11. Ten \$1000 bonds at $73\frac{3}{4}$, brokerage \$2.50 per bond.

12. Which yields the greater rate of income: 4% stock at $78\frac{1}{2}$ or 5% bonds at 112? How much greater?

13. In order to receive \$63 every 3 mo., how much money must I invest at $4\frac{1}{2}\%$ annually?

14. Make and work a problem showing the per cent of income from an investment in real estate. Consider taxes, insurance, repairs, and interest on a mortgage.

1. Potatoes are priced at 80¢ a bushel or 25¢ a peck. How much is saved by buying a bushel at a time?

2. Mr. Anderson bought a cream separator priced at $76.50 cash for $7 down and $7 a month for 11 mo. How much more was the installment price than the cash price?

3. At a sale, Mrs. Anderson bought 6 sheets at 20% off the regular price of $1.50 each. How much did she save?

4. Mr. Foster's store is lighted by twenty 150-watt bulbs. At 6¢ per kw-hr, how much does it cost per hour to light the store?

5. Make out a bill for 5 items which a family might buy at a grocery store. Receipt the bill.

6. The April reading of our gas meter was 64,800 cu. ft., and the May reading was 67,300 cu. ft. The charge was $1.30 for the first 1000 cu. ft. and $1.10 per 1000 cu. ft. for the balance. Find the amount of the bill.

7. Make a reasonable budget for a monthly income of $180, allowing for food, clothing, shelter, operating expenses, other expenses, and savings.

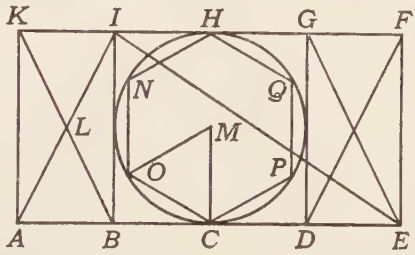
8. The Endicotts borrowed $4000 from a building and loan association, taking out twenty $200 shares at $1.00 each per month to cover the loan. Find their monthly payment, including interest on the loan at 6%.

9. One year, the Kents paid $56 fire insurance, $144 taxes, and $126 repairs on their home. Find the average cost per month.

10. The Browns bought a new home for $6000, paying $2000 down, and making monthly payments of $60 on the balance, including 6% interest. What was the balance due on the principal at the end of 4 mo.?

1. Draw a parallelogram with base $2''$ and slant sides $\frac{3}{4}''$.

2. Give the name of each of the following figures in this drawing. If the figure is a triangle, name the kind of triangle. *DEF, AEI, OCM, BDGI,* the straight-line figure *CPQHNO,* the curve *CPQHNO, ABIK, ADFI,* the curve *OH, AEGI.*



3. Find the perimeter of *ABIK; BDGI; OCM.*

4. Find the circumference of the circle in the drawing.

5. Give the formula for finding the area of *ABK; BEFI; BDGI;* curve *CPQHNO; ADFI; AEGI.* If the figures are drawn to the scale $1'' = 6'$, what is the actual area of each figure?

6. (a) On a scale of $1'' = 1'$, construct a triangle which has sides $2'$, $3'$, and $4'$ long. (b) Bisect one angle of the triangle. (c) Erect a perpendicular at the middle point of one side. (d) Using a protractor, measure the three angles.

7. How many cubic feet of air per person are there in an office 12 ft. in width, 15 ft. in height, and 40 ft. in length, in which 6 people work?

8. Find the volume and the lateral area of a cylinder having a radius of $14''$ and a height of $20''$.

***9.** Two towns in Belgium are 120 kilometers apart. How far apart are they in miles? (1 km. $= \frac{5}{8}$ mi.)

***10.** Some prize apples are piled in a pyramid that is 5 ft. square at the base and 4 ft. high. How many bushels of apples are in the pile? (1 bu. $= 1\frac{1}{4}$ cu. ft.)

*Optional material.

Make an equation for each of the following; then solve and check your equation:

1. One third of a number equals 9.
2. The product of 7 and a number equals 63.
3. Sixteen equals the sum of 11 and a number.

Solve each of the following equations:

4. $x + 17 = 24$
5. $15 = y - 8$
6. $15n = 65$
7. $\frac{x}{3} = .8$
8. $\frac{2y}{5} = 16$
9. $\frac{3m}{8} + 6 = 12$
10. $5x - 3x + x = .36$
11. $\frac{2}{3}r + 1\frac{1}{3}r + 2r = 18$

Find the unknown quantity in each of the following proportions, and check:

12. $\frac{x}{12} = \frac{21}{36}$
13. $\frac{15}{18} = \frac{x}{24}$
14. $\frac{5}{x} = \frac{4}{28}$
15. $\frac{3.6}{2.4} = \frac{6}{y}$
16. $x : 14 = 56 : 7$
17. $36 : 27 : : x : 42$

18. The sum of a certain number and 16 is 52. Write this as an equation, and find the number.

19. If Mr. Alden can set out 450 tomato plants in 3 hr., how long will it take him to set out 2400 plants?

20. Divide \$300 among A, B, and C so that A shall have twice as much as B; and C, \$20 more than B.

21. A rectangle whose perimeter is 180 ft. is $\frac{2}{3}$ as wide as it is long. Find its dimensions.

22. Use the formula $V = lwh$ to find the value of w if $V = 192$, $l = 8$, and $h = 4$.

*23. Change the formula $C = 2\pi r$ to a formula for finding r. Find the value of r when $C = 88$.

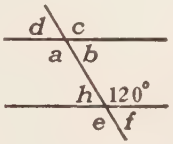
*Optional material.

1. Draw a symmetrical plane figure; a symmetrical solid.

2. Draw two congruent rectangles; two congruent triangles.

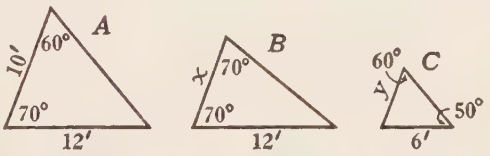
3. Which of the following figures are equivalent: a 4″ square, a rectangle 5″ by 3″, and a triangle having a base of 8″ and a height of 4″?

4. Give the number of degrees in each of the lettered angles in this figure.

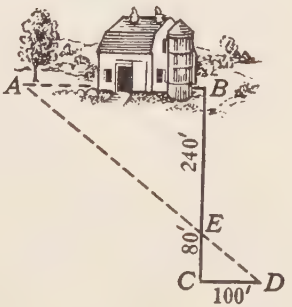


5. The shadow of a flagpole is 20 ft. long when the shadow of a 6-ft. post is 4 ft. long. How high is the flagpole?

6. Which of these three triangles are similar? In the triangle similar to A, the side marked with a letter equals ? ft.



7. Find the actual distance AB in the drawing at the right.



*8. Two boys started walking from a street corner. After one had gone 120 ft. east and the other 160 ft. south, what was the shortest distance between them?

9. Measure a distance AB along one side of a road, and mark points A and B. From B, sight some object C directly opposite B on the other side of the road. From A, measure $\angle BAC$ with a transit. By means of a scale drawing, find the distance across the road. Check this distance by measuring directly.

*Optional material.

1. Name and define six terms used in addition, subtraction, multiplication, and division.

2. In what table or tables of measures does each of these units belong: A., gro., pk., gal., qt., T., rd., mi., min., cwt., hr., cu. ft., sq. yd.?

3. Give and define six terms used in: (a) percentage and its business applications; (b) interest and banking; (c) insurance; (d) taxes; (e) investments.

4. Explain the difference between:

   a. Rate of profit and rate of interest.

   b. Trade discount and bank discount.

   c. An annuity policy and an endowment policy.

5. Describe or define each of the following: Statement, check, sales slip, deposit slip, promissory note, trade acceptance, certified check, share of stock, bond, mortgage, money order, insurance policy.

6. Draw and label (a) four kinds of angles; (b) four kinds of triangles.

7. Draw and label a rectangle; a hexagon; an octagon; two similar triangles; a trapezoid.

8. Name and draw four kinds of graphs.

9. What is a formula? Name three common formulas, and tell for what each is used.

10. Explain the difference between symmetrical, similar, equivalent, and congruent figures.

11. Define an equation; the members of an equation; the terms of an equation; a ratio; a proportion. Give examples of each.

MEASURE OF LENGTH

12 in. (″) = 1 ft. (′)

3 ft. (′), 36 in. = 1 yd.

5½ yd., 16½ ft. = 1 rd.

320 rd., 5280 ft. = 1 mi.

MEASURE OF WEIGHT

16 oz. = 1 lb.
100 lb. = 1 cwt.
2000 lb. = 1 T.

LIQUID MEASURE

2 cupfuls = 1 pt.
2 pt. = 1 qt.
4 qt. = 1 gal.

DRY MEASURE

2 pt. = 1 qt.
8 qt. = 1 pk.
4 pk., 32 qt. = 1 bu.

COUNTING

12 things = 1 doz.
12 doz., 144 things = 1 gross

MEASURE OF TIME

60 sec. = 1 min.
60 min. = 1 hr.
24 hr. = 1 da.
7 da. = 1 wk.
52 wk., 12 mo., 365 da. = 1 yr.
366 da. = 1 leap year
100 yr. = 1 century

SURFACE MEASURE

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
30¼ sq. yd. = 1 sq. rd.
160 sq. rd. = 1 A.
640 A. = 1 sq. mi.

CUBIC MEASURE

1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.

CIRCULAR MEASURE

60 sec. (″) = 1 min. (′)
60 min. (′) = 1 degree (°)
90 degrees = a right angle
180 degrees = a straight angle
360 degrees in a circle

USEFUL FACTS ABOUT MEASURES

1 gal. = 231 cu. in.
1 gal. water = 8.35 lb.
7½ gal. = 1 cu. ft.
31½ gal. = 1 bbl.
1 cu. ft. water = about 62½ lb.
1 cu. ft. ice = 57.2 lb.
.8 bu. = 1 cu. ft.
1 bu. = 1¼ cu. ft. or about 2150 cu. in.
1 bu. wheat = 60 lb.
1 bu. potatoes = 60 lb. in most states
1 bbl. flour = 196 lb.
1 T. hay = 500 cu. ft.
1 T. hard coal = about 35 cu. ft.

LUMBER MEASURE

A board 1″ thick, 1′ wide, and 1′ long = 1 board foot (bd.ft.)
1 cord of wood = 128 cu. ft.

METRIC TABLE OF LENGTH

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters (cm.) = 1 decimeter (dm.)
10 decimeters (dm.) = 1 meter (m.)
10 meters (m.) = 1 dekameter (dkm.)
10 dekameters (dkm.) = 1 hektometer (hm.)
10 hektometers (hm.) = 1 kilometer (km.)

COMMON METRIC UNITS OF CAPACITY

1 cubic decimeter (cu.dm.) = 1 liter (l.)

METRIC EQUIVALENTS

1 centimeter = .39 in.
1 meter = 39.37 in. / 1.1 yd.
1 kilometer = $\frac{5}{8}$ mi.
1 liter = 1.1 qt.
1 gram = .035 oz.
1 kilogram = 2.2 lb.
1 metric ton = 1.1 T.
1 square meter = 1.2 sq. yd.
1 cubic meter = 1.31 cu. yd.

COMMON METRIC UNITS OF WEIGHT

1000 grams (g.) = 1 kilogram (kg.)
1000 kilograms (kg.) = 1 metric ton

PAPER MEASURE

24 or 25 sheets = 1 quire
20 quires = 1 ream

KITCHEN MEASURE

3 teaspoonfuls = 1 tablespoonful
12 tablespoonfuls = 1 cup
2 cupfuls = 1 pt.

MARINERS' MEASURE

6 ft. = 1 fathom
120 fathoms = 1 cable length
5280 ft. = 1 land mile
6080 ft. = 1 nautical mile
1.152 land mi. or 1 nautical mi. = 1 knot
3 knots = 1 league

FORMULAS

PLANE FIGURES

Circle: $C = 2\pi r$ or πd; $d = \frac{C}{\pi}$; $A = \pi r^2$

Square: $A = s^2$

Rectangle: $A = lw$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}(b1 + b2)h$ or $\frac{(b1 + b2)h}{2}$

Triangle: $A = \frac{1}{2}bh$ or $\frac{bh}{2}$

SOLID FIGURES

Rectangular solid: $V = lwh$

Prism: $V = Bh$

Cube: $V = e^3$

Cylinder (lateral surface): $V = Bh$ or $\pi r^2 h$; $A = 2\pi rh$ or πdh

Cone: $V = \frac{1}{3}Bh$ or $\frac{1}{3}\pi r^2 h$

Pyramid: $V = \frac{1}{3}Bh$

Sphere: $V = \frac{4}{3}\pi r^3$; $A = 4\pi r^2$

MISCELLANEOUS

Distance: $d = rt$ Interest: $i = prt$
Cost: $C = np$ Percentage: $p = br$
Hypotenuse rule: $c^2 = a^2 + b^2$

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